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ALL TIGHT DESCRIPTIONS OF 3-STARS IN 3-POLYTOPES WITH GIRTH 5

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Abstract

Lebesgue (1940) proved that every 3-polytope P_5 of girth 5 has a path of three vertices of degree 3. Madaras (2004) refined this by showing that every P_5 has a 3-vertex with two 3-neighbors and the third neighbor of degree at most 4. This description of 3-stars in P_5s is tight in the sense that no its parameter can be strengthened due to the dodecahedron combined with the existence of a P_5 in which every 3-vertex has a 4-neighbor.

We give another tight description of 3-stars in P_5s: there is a vertex of degree at most 4 having three 3-neighbors. Furthermore, we show that there are only these two tight descriptions of 3-stars in P_5s.

Also, we give a tight description of stars with at least three rays in P_5s and pose a problem of describing all such descriptions. Finally, we prove a structural theorem about P_5s that might be useful in further research.

Keywords: 3-polytope, planar graph, structure properties, k-star.

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