

A NOTE ON THE TOTAL DETECTION NUMBERS OF CYCLES

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Abstract

Let G be a connected graph of size at least 2 and $c : E(G) \rightarrow \{0, 1, \dots, k-1\}$ an edge coloring (or labeling) of G using k labels, where adjacent edges may be assigned the same label. For each vertex v of G , the color code of v with respect to c is the k -vector $\text{code}(v) = (a_0, a_1, \dots, a_{k-1})$, where a_i is the number of edges incident with v that are labeled i for $0 \leq i \leq k-1$. The labeling c is called a detectable labeling if distinct vertices in G have distinct color codes. The value $\text{val}(c)$ of a detectable labeling c of a graph G is the sum of the labels assigned to the edges in G . The total detection number $\text{td}(G)$ of G is defined by $\text{td}(G) = \min\{\text{val}(c)\}$, where the minimum is taken over all detectable labelings c of G . We investigate the problem of determining the total detection numbers of cycles.

Keywords: vertex-distinguishing coloring, detectable labeling, detection number, total detection number, Hamiltonian graph.

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