

## ON $\bullet$ -LINE SIGNED GRAPHS $L_{\bullet}(S)$

DEEPA SINHA

*South Asian University  
Akbar Bhawan, Chanakyapuri  
New Delhi-110 021, India*

**e-mail:** deepa\_sinha2001@yahoo.com

AND

AYUSHI DHAMA

*Centre for Mathematical Sciences  
Banasthali University  
Banasthali-304 022 Rajasthan, India*

**e-mail:** ayushi.dhama2@gmail.com

### Abstract

A *signed graph* (or *sigraph* for short) is an ordered pair  $S = (S^u, \sigma)$ , where  $S^u$  is a graph,  $G = (V, E)$ , called the underlying graph of  $S$  and  $\sigma : E \rightarrow \{+, -\}$  is a function from the edge set  $E$  of  $S^u$  into the set  $\{+, -\}$ . For a sigraph  $S$  its  $\bullet$ -line sigraph,  $L_{\bullet}(S)$  is the sigraph in which the edges of  $S$  are represented as vertices, two of these vertices are defined adjacent whenever the corresponding edges in  $S$  have a vertex in common, any such  $L$ -edge  $ee'$  has the sign given by the product of the signs of the edges incident with the vertex in  $e \cap e'$ . In this paper we establish a structural characterization of  $\bullet$ -line sigraphs, extending a well known characterization of line graphs due to Harary. Further we study several standard properties of  $\bullet$ -line sigraphs, such as the balanced  $\bullet$ -line sigraphs, sign-compatible  $\bullet$ -line sigraphs and  $\mathcal{C}$ -sign-compatible  $\bullet$ -line sigraphs.

**Keywords:** sigraph, line graph,  $\bullet$ -line sigraph, balance, sign-compatibility,  $\mathcal{C}$ -sign-compatibility.

**2010 Mathematics Subject Classification:** 05C22, 05C75.

### REFERENCES

- [1] B.D. Acharya, *Signed intersection graphs*, J. Discrete Math. Sci. Cryptogr. **13** (2010) 553–569.  
doi:10.1080/09720529.2010.10698314

- [2] M. Acharya and D. Sinha, *Characterizations of line sigraphs*, Nat. Acad. Sci. Lett. **28** (2005) 31–34.  
Extended abstract in: Electron. Notes Discrete Math. **15** (2003) 12.
- [3] M. Behzad and G.T. Chartrand, *Line coloring of signed graphs*, Elem. Math. **24(3)** (1969) 49–52.
- [4] L.W. Beineke, *Derived graphs and digraphs*, in: Beiträge zur Graphentheorie, H. Sachs, H. Voss and H. Walter (Ed(s)), (Teubner, Leipzig, 1968) 17–33.
- [5] L.W. Beineke, *Characterizations of derived graphs*, J. Combin. Theory (B) **9** (1970) 129–135.  
doi:10.1016/S0021-9800(70)80019-9
- [6] M.K. Gill, Contribution to some topics in graph theory and its applications (Ph.D. Thesis, Indian Institute of Technology, Bombay, 1983).
- [7] F. Harary, *On the notion of balance of a signed graph*, Michigan Math. J. **2** (1953) 143–146.  
doi:10.1307/mmj/1028989917
- [8] F. Harary, Graph Theory (Addison-Wesley Publ. Comp., Reading, Massachusetts, 1969).
- [9] F. Harary and R.Z. Norman, *Some properties of line digraphs*, Rend. Circ. Mat. Palermo (2) Suppl. **9** (1960) 161–168.
- [10] R.L. Hemminger and L.W. Beineke, *Line graphs and line digraphs*, in: Selected Topics in Graph Theory, L.W. Beineke and R.J. Wilson (Ed(s)), (Academic Press Inc., 1978) 271–305.
- [11] J. Krausz, *Démonstration nouvelle d’une théorème de Whitney sur les réseaux*, Mat. Fiz. Lapok **50** (1943) 75–89.
- [12] V.V. Menon, *On repeated interchange graphs*, Amer. Math. Monthly **73** (1966) 986–989.  
doi:10.2307/2314503
- [13] O. Ore, Theory of Graphs (Amer. Math. Soc. Colloq. Publ. 38, Providence, 1962).
- [14] G. Sabidussi, *Graph derivatives*, Math. Z. **76** (1961) 385–401.  
doi:10.1007/BF01210984
- [15] E. Sampathkumar, *Point-signed and line-signed graphs*, Karnatak Univ. Graph Theory Res. Rep. No.1 (1973) (also see Abstract No. 1 in: Graph Theory Newsletter **2(2)** (1972), National Academy Science Letters **7** (1984) 91–93).
- [16] D. Sinha, New frontiers in the theory of signed graph (Ph.D. Thesis, University of Delhi, Faculty of Technology, 2005).
- [17] D. Sinha and A. Dhama, *Sign-compatibility of some derived signed graphs*, Indian J. Math. **55** (2013) 23–40.
- [18] D. Sinha and A. Dhama, *Canonical-sign-compatibility of some signed graphs*, J. Combin. Inf. Syst. Sci. **38** (2013) 129–138.

- [19] D.B. West, *Introduction to Graph Theory* (Prentice-Hall of India Pvt. Ltd., 1996).
- [20] H. Whitney, *Congruent graphs and the connectivity of graphs*, Amer. J. Math. **54** (1932) 150–168.  
doi:10.2307/2371086
- [21] T. Zaslavsky, *A mathematical bibliography of signed and gain graphs and allied areas*, 7th Edition, Electron. J. Combin. (1998) #DS8.
- [22] T. Zaslavsky, *Glossary of signed and gain graphs and allied areas*, Second Edition, Electron. J. Combin. (1998) #DS9.
- [23] T. Zaslavsky, *Signed analogs of bipartite graphs*, Discrete Math. **179** (1998) 205–216.  
doi:10.1016/S0012-365X(96)00386-X

Received 25 September 2013

Revised 7 April 2014

Accepted 9 April 2014