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A NOTE ON NON-DOMINATING SET PARTITIONS IN GRAPHS

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Abstract

A set S of vertices of a graph G is a dominating set if every vertex not in S is adjacent to a vertex of S and is a total dominating set if every vertex of G is adjacent to a vertex of S. The cardinality of a minimum dominating (total dominating) set of G is called the domination (total domination) number. A set that does not dominate (totally dominate) G is called a non-dominating (non-total dominating) set of G. A partition of the vertices of G into non-dominating (non-total dominating) sets is a non-dominating (non-total dominating) set partition. We show that the minimum number of sets in a non-dominating set partition of a graph G equals the total domination number of its complement $\overline{G}$ and the minimum number of sets in a non-total dominating set partition of G equals the domination number of $\overline{G}$. This perspective yields new upper bounds on the domination and total domination numbers. We motivate the study of these concepts with a social network application.

Keywords: domination, total domination, non-dominating partition, non-total dominating partition.

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