

## RADIO GRACEFUL HAMMING GRAPHS

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### Abstract

For  $k \in \mathbb{Z}_+$  and  $G$  a simple, connected graph, a  $k$ -radio labeling  $f : V(G) \rightarrow \mathbb{Z}_+$  of  $G$  requires all pairs of distinct vertices  $u$  and  $v$  to satisfy  $|f(u) - f(v)| \geq k + 1 - d(u, v)$ . We consider  $k$ -radio labelings of  $G$  when  $k = \text{diam}(G)$ . In this setting,  $f$  is injective; if  $f$  is also surjective onto  $\{1, 2, \dots, |V(G)|\}$ , then  $f$  is a *consecutive radio labeling*. Graphs that can be labeled with such a labeling are called *radio graceful*. In this paper, we give two results on the existence of radio graceful Hamming graphs. The main result shows that the Cartesian product of  $t$  copies of a complete graph is radio graceful for certain  $t$ . Graphs of this form provide infinitely many examples of radio graceful graphs of arbitrary diameter. We also show that these graphs are not radio graceful for large  $t$ .

**Keywords:** radio labeling, radio graceful graph, Hamming graph.

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