

## VERTEX COLORINGS WITHOUT RAINBOW SUBGRAPHS

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### Abstract

Given a coloring of the vertices of a graph  $G$ , we say a subgraph is rainbow if its vertices receive distinct colors. For a graph  $F$ , we define the  $F$ -upper chromatic number of  $G$  as the maximum number of colors that can be used to color the vertices of  $G$  such that there is no rainbow copy of  $F$ . We present some results on this parameter for certain graph classes. The focus is on the case that  $F$  is a star or triangle. For example, we show that the  $K_3$ -upper chromatic number of any maximal outerplanar graph on  $n$  vertices is  $\lfloor n/2 \rfloor + 1$ .

**Keywords:** coloring, rainbow, monochromatic, forbidden, path.

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