Discussiones Mathematicae
Graph Theory 36 (2016) 977–988
doi:10.7151/dmgt.1915

SHARP UPPER BOUNDS ON THE SIGNLESS LAPLACIAN SPECTRAL RADIUS OF STRONGLY CONNECTED DIGRAPHS[1]

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Abstract

Let $G = (V(G), E(G))$ be a simple strongly connected digraph and $q(G)$ be the signless Laplacian spectral radius of G. For any vertex $v_i \in V(G)$, let d_i^+ denote the outdegree of v_i, m_i^+ denote the average 2-outdegree of v_i, and N_i^+ denote the set of out-neighbors of v_i. In this paper, we prove that:

(1) $q(G) = d_1^+ + d_2^+$, $(d_1^+ \neq d_2^+)$ if and only if G is a star digraph $\overleftrightarrow{K}_{1,n-1}$, where d_1^+, d_2^+ are the maximum and the second maximum outdegree, respectively ($\overleftrightarrow{K}_{1,n-1}$ is the digraph on n vertices obtained from a star graph $K_{1,n-1}$ by replacing each edge with a pair of oppositely directed arcs).

(2) $q(G) \leq \max\left\{\frac{1}{2}\left(d_i^+ + \sqrt{{d_i^+}^2 + 8d_i^+ m_i^+}\right) : v_i \in V(G)\right\}$ with equality if and only if G is a regular digraph.

(3) $q(G) \leq \max\left\{\frac{1}{2}\left(d_i^+ + \sqrt{{d_i^+}^2 + \frac{4}{d_i^+}\sum_{v_j \in N_i^+} d_j^+(d_j^+ + m_j^+)}\right) : v_i \in V(G)\right\}$. Moreover, the equality holds if and only if G is a regular digraph or a bipartite semiregular digraph.

(4) $q(G) \leq \max\left\{\frac{1}{2}\left(d_i^+ + 2d_j^+ - 1 + \sqrt{(d_i^+ - 2d_j^+ + 1)^2 + 4d_i^+}\right) : (v_j, v_i) \in E(G)\right\}$. If the equality holds, then G is a regular digraph or $G \in \Omega$, where Ω is a class of digraphs defined in this paper.

Keywords: digraph, signless Laplacian spectral radius.

2010 Mathematics Subject Classification: 05C50, 15A18.

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[1]The research was supported by the National Natural Science Foundation of China (Grant No. 11171273).

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Received 26 March 2015
Revised 13 January 2016
Accepted 13 January 2016