

BOUNDS FOR THE b -CHROMATIC NUMBER OF SUBGRAPHS AND EDGE-DELETED SUBGRAPHS

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Abstract

A b -coloring of a graph G with k colors is a proper coloring of G using k colors in which each color class contains a color dominating vertex, that is, a vertex which has a neighbor in each of the other color classes. The largest positive integer k for which G has a b -coloring using k colors is the b -chromatic number $b(G)$ of G . In this paper, we obtain bounds for the b -chromatic number of induced subgraphs in terms of the b -chromatic number of the original graph. This turns out to be a generalization of the result due to R. Balakrishnan *et al.* [*Bounds for the b -chromatic number of $G - v$* , *Discrete Appl. Math.* 161 (2013) 1173–1179]. Also we show that for any connected graph G and any $e \in E(G)$, $b(G - e) \leq b(G) + \lceil \frac{n}{2} \rceil - 2$. Further, we determine all graphs which attain the upper bound. Finally, we conclude by finding bound for the b -chromatic number of any subgraph.

Keywords: b -coloring, b -chromatic number.

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