

## GRAPHS WITH LARGE GENERALIZED (EDGE-)CONNECTIVITY

XUELIANG LI

*Center for Combinatorics and LPMC*  
*Nankai University*  
*Tianjin 300071, China*

**e-mail:** lxl@nankai.edu.cn

AND

YAPING MAO

*Department of Mathematics*  
*Qinghai Normal University*  
*Qinghai 810008, China*

**e-mail:** maoyaping@ymail.com

### Abstract

The generalized  $k$ -connectivity  $\kappa_k(G)$  of a graph  $G$ , introduced by Hager in 1985, is a nice generalization of the classical connectivity. Recently, as a natural counterpart, we proposed the concept of generalized  $k$ -edge-connectivity  $\lambda_k(G)$ . In this paper, graphs of order  $n$  such that  $\kappa_k(G) = n - \frac{k}{2} - 1$  and  $\lambda_k(G) = n - \frac{k}{2} - 1$  for even  $k$  are characterized.

**Keywords:** (edge-)connectivity, Steiner tree, internally disjoint trees, edge-disjoint trees, packing, generalized (edge-)connectivity.

**2010 Mathematics Subject Classification:** 05C40, 05C05, 05C70, 05C75.

### REFERENCES

- [1] L.W. Beineke and R.J. Wilson, *Topics in Structural Graph Theory* (Cambridge University Press, 2013).
- [2] F.T. Boesch and S. Chen, *A generalization of line connectivity and optimally invulnerable graphs*, *SIAM J. Appl. Math.* **34** (1978) 657–665.  
doi:10.1137/0134052
- [3] J.A. Bondy and U.S.R. Murty, *Graph Theory* (GTM 244, Springer, 2008).

- [4] G. Chartrand, S.F. Kapoor, L. Lesniak and D.R. Lick, *Generalized connectivity in graphs*, Bull. Bombay Math. Colloq. **2** (1984) 1–6.
- [5] G. Chartrand, F. Okamoto and P. Zhang, *Rainbow trees in graphs and generalized connectivity*, Networks **55** (2010) 360–367.
- [6] D.P. Day, O.R. Oellermann and H.C. Swart, *The  $\ell$ -connectivity function of trees and complete multipartite graphs*, J. Combin. Math. Combin. Comput. **10** (1991) 183–192.
- [7] D.L. Goldsmith, *On the second-order edge-connectivity of a graph*, Congr. Numer. **29** (1980) 479–484.
- [8] D.L. Goldsmith, *On the  $n$ th order connectivity of a graph*, Congr. Numer. **32** (1981) 375–382.
- [9] D.L. Goldsmith, B. Manval and V. Faber, *Seperation of graphs into three components by removal of edges*, J. Graph Theory **4** (1980) 213–218.  
doi:10.1002/jgt.3190040209
- [10] M. Grötschel, *The Steiner tree packing problem in VLSI design*, Math. Program. **78** (1997) 265–281.  
doi:10.1007/BF02614374
- [11] M. Grötschel, A. Martin and R. Weismantel, *Packing Steiner trees: A cutting plane algorithm and commputational results*, Math. Program. **72** (1996) 125–145.
- [12] M. Hager, *Pendant tree-connectivity*, J. Combin. Theory **38** (1985) 179–189.  
doi:10.1016/0095-8956(85)90083-8
- [13] M. Hager, *Path-connectivity in graphs*, Discrete Math. **59** (1986) 53–59.  
doi:10.1016/0012-365X(86)90068-3
- [14] H.R. Hind, O.R. Oellermann, *Menger-type results for three or more vertices*, Congr. Numer. **113** (1996) 179–204.
- [15] K. Jain, M. Mahdian and M. Salavatipour, *Packing Steiner trees*, in: Proc. 14th ACM-SIAM Symposium on Discrete Algorithms (Baltimore, 2003) 266–274.
- [16] M. Kriesell, *Edge disjoint trees containing some given vertices in a graph*, J. Combin. Theory Ser. B **88** (2003) 53–65.  
doi:10.1016/S0095-8956(02)00013-8
- [17] M. Kriesell, *Edge-disjoint Steiner trees in graphs without large bridges*, J. Graph Theory **62** (2009) 188–198.  
doi:10.1002/jgt.20389
- [18] L. Lau, *An approximate max-Steiner-tree-packing min-Steiner-cut theorem*, Combinatorica **27** (2007) 71–90.  
doi:10.1002/jgt.20389
- [19] W. Mader, *Über die maximalzahl kantendisjunkter  $A$ -wege*, Arch. Math. **30** (1978) 325–336.  
doi:10.1007/BF01226062

- [20] W. Mader, *Über die maximalzahl kreuzungsfreier  $H$ -wege*, Arch. Math. **31** (1978) 387–402.  
doi:10.1007/BF01226465
- [21] H. Li, X. Li and Y. Mao, *On extremal graphs with at most two internally disjoint Steiner trees connecting any three vertices*, Bull. Malays. Math. Sci. Soc. **37** (2014) 747–756.
- [22] H. Li, X. Li, Y. Mao and Y. Sun, *Note on the generalized connectivity*, Ars Combin. **114** (2014) 193–202.
- [23] H. Li, X. Li, Y. Mao and J. Yue, *Note on the spanning-tree packing number of lexicographic product graphs*, Discrete Math. **338** (2015) 669–673.  
doi:10.1016/j.disc.2014.12.007
- [24] H. Li, X. Li and Y. Sun, *The generalized 3-connectivity of Cartesian product graphs*, Discrete Math. Theor. Comput. Sci. **14** (2012) 43–54.  
doi:10.1016/j.commsci.2011.09.003
- [25] S. Li, W. Li and X. Li, *The generalized connectivity of complete equipartition 3-partite graphs*, Bull. Malays. Math. Sci. Soc. **37** (2014) 103–121.
- [26] S. Li and X. Li, *Note on the hardness of generalized connectivity*, J. Comb. Optim. **24** (2012) 389–396.  
doi:10.1007/s10878-011-9399-x
- [27] S. Li, X. Li and W. Zhou, *Sharp bounds for the generalized connectivity  $\kappa_3(G)$* , Discrete Math. **310** (2010) 2147–2163.  
doi:10.1016/j.disc.2010.04.011
- [28] X. Li and Y. Mao, *Nordhaus-Gaddum-type results for the generalized edge-connectivity of graphs*, Discrete Appl. Math. **185** (2015) 102–112.  
doi:10.1016/j.dam.2014.12.009
- [29] X. Li and Y. Mao, *On extremal graphs with at most  $\ell$  internally disjoint Steiner trees connecting any  $n - 1$  vertices*, Graphs Combin. **31** (2015) 2231–2259.  
doi:10.1007/s00373-014-1500-7
- [30] X. Li and Y. Mao, *The generalized 3-connectivity of lexicographical product graphs*, Discrete Math. Theor. Comput. Sci. **16** (2014) 339–354.
- [31] X. Li and Y. Mao, *The minimal size of a graph with given generalized 3-edge-connectivity*, Ars Combin. **118** (2015) 63–72.
- [32] X. Li, Y. Mao and Y. Sun, *On the generalized (edge-)connectivity of graphs*, Australas. J. Combin. **58** (2014) 304–319.
- [33] C.St.J.A. Nash-Williams, *Edge-disjoint spanning trees of finite graphs*, J. Lond. Math. Soc. **36** (1961) 445–450.  
doi:10.1112/jlms/s1-36.1.445
- [34] O.R. Oellermann, *Connectivity and edge-connectivity in graphs: A survey*, Congr. Numer. **116** (1996) 231–252.

- [35] O.R. Oellermann, *On the  $\ell$ -connectivity of a graph*, *Graphs Combin.* **3** (1987) 285–299.  
doi:10.1007/BF01788551
- [36] O.R. Oellermann, *A note on the  $\ell$ -connectivity function of a graph*, *Congr. Numer.* **60** (1987) 181–188.
- [37] F. Okamoto and P. Zhang, *The tree connectivity of regular complete bipartite graphs*, *J. Combin. Math. Combin. Comput.* **74** (2010) 279–293.
- [38] N.A. Sherwani, *Algorithms for VLSI Physical Design Automation*, 3rd Edition (Kluwer Acad. Pub., Dordrecht, 1999).
- [39] W. Tutte, *On the problem of decomposing a graph into  $n$  connected factors*, *J. Lond. Math. Soc.* **36** (1961) 221–230.  
doi:10.1112/jlms/s1-36.1.221
- [40] D. West and H. Wu, *Packing Steiner trees and  $S$ -connectors in graphs*, *J. Combin. Theory Ser. B* **102** (2012) 186–205.

Received 14 December 2014

Revised 7 January 2016

Accepted 7 January 2016



