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THE DYNAMICS OF THE FOREST GRAPH OPERATOR

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Abstract

In 1966, Cummins introduced the "tree graph": the tree graph $\mathbf{T}(G)$ of a graph G (possibly infinite) has all its spanning trees as vertices, and distinct such trees correspond to adjacent vertices if they differ in just one edge, i.e., two spanning trees T_1 and T_2 are adjacent if $T_2 = T_1 - e + f$ for some edges $e \in T_1$ and $f \notin T_1$. The tree graph of a connected graph need not be connected. To obviate this difficulty we define the "forest graph": let G be a labeled graph of order α, finite or infinite, and let $\mathfrak{N}(G)$ be the set of all labeled maximal forests of G. The forest graph of G, denoted by $\mathbf{F}(G)$, is the graph with vertex set $\mathfrak{N}(G)$ in which two maximal forests F_1, F_2 of G form an edge if and only if they differ exactly by one edge, i.e., $F_2 = F_1 - e + f$ for some edges $e \in F_1$ and $f \notin F_1$.

Using the theory of cardinal numbers, Zorn's lemma, transfinite induction, the axiom of choice and the well-ordering principle, we determine the **F**-convergence, **F**-divergence, **F**-depth and **F**-stability of any graph G. In particular it is shown that a graph G (finite or infinite) is **F**-convergent if and only if G has at most one cycle of length 3. The **F**-stable graphs are precisely K_3 and K_1. The **F**-depth of any graph G different from K_3 and K_1 is finite. We also determine various parameters of $\mathbf{F}(G)$ for an infinite graph G, including the number, order, size, and degree of its components.

Keywords: forest graph operator, graph dynamics.

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