

THE GUTMAN INDEX AND THE EDGE-WIENER INDEX OF GRAPHS WITH GIVEN VERTEX-CONNECTIVITY

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Abstract

The Gutman index and the edge-Wiener index have been extensively investigated particularly in the last decade. An important stream of research on graph indices is to bound indices in terms of the order and other parameters of given graph. In this paper we present asymptotically sharp upper bounds on the Gutman index and the edge-Wiener index for graphs of given order and vertex-connectivity κ , where κ is a constant. Our results substantially generalize and extend known results in the area.

Keywords: Gutman index, edge-Wiener index, vertex-connectivity.

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