

INTEGRAL CAYLEY SUM GRAPHS AND GROUPS

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Abstract

For any positive integer k , let \mathcal{A}_k denote the set of finite abelian groups G such that for any subgroup H of G all Cayley sum graphs $\text{CayS}(H, S)$ are integral if $|S| = k$. A finite abelian group G is called Cayley sum integral if for any subgroup H of G all Cayley sum graphs on H are integral. In this paper, the classes \mathcal{A}_2 and \mathcal{A}_3 are classified. As an application, we determine all finite Cayley sum integral groups.

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