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COMPUTING THE METRIC DIMENSION OF A GRAPH FROM PRIMARY SUBGRAPHS

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Abstract

Let G be a connected graph. Given an ordered set $W = \{w_1, \ldots, w_k\} \subseteq V(G)$ and a vertex $u \in V(G)$, the representation of u with respect to W is the ordered k-tuple $(d(u, w_1), d(u, w_2), \ldots, d(u, w_k))$, where $d(u, w_i)$ denotes the distance between u and w_i. The set W is a metric generator for G if every two different vertices of G have distinct representations. A minimum cardinality metric generator is called a *metric basis* of G and its cardinality is called the *metric dimension* of G. It is well known that the problem of finding the metric dimension of a graph is NP-hard. In this paper we obtain closed formulae for the metric dimension of graphs with cut vertices. The main results are applied to specific constructions including rooted product graphs, corona product graphs, block graphs and chains of graphs.

Keywords: metric dimension, metric basis, primary subgraphs, rooted product graphs, corona product graphs.

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