

THE DICHROMATIC NUMBER OF INFINITE FAMILIES OF CIRCULANT TOURNAMENTS

NAHID JAVIER AND BERNARDO LLANO

*Departamento de Matemáticas
Universidad Autónoma Metropolitana Iztapalapa
San Rafael Atlixco 186, Colonia Vicentina
09340, México, D.F., Mexico*

e-mail: {nahid,llano}@xanum.uam.mx

Abstract

The *dichromatic number* $dc(D)$ of a digraph D is defined to be the minimum number of colors such that the vertices of D can be colored in such a way that every chromatic class induces an acyclic subdigraph in D . The *cyclic circulant tournament* is denoted by $T = \vec{C}_{2n+1}(1, 2, \dots, n)$, where $V(T) = \mathbb{Z}_{2n+1}$ and for every jump $j \in \{1, 2, \dots, n\}$ there exist the arcs $(a, a + j)$ for every $a \in \mathbb{Z}_{2n+1}$. Consider the circulant tournament $\vec{C}_{2n+1}\langle k \rangle$ obtained from the cyclic tournament by reversing one of its jumps, that is, $\vec{C}_{2n+1}\langle k \rangle$ has the same arc set as $\vec{C}_{2n+1}(1, 2, \dots, n)$ except for $j = k$ in which case, the arcs are $(a, a - k)$ for every $a \in \mathbb{Z}_{2n+1}$. In this paper, we prove that $dc(\vec{C}_{2n+1}\langle k \rangle) \in \{2, 3, 4\}$ for every $k \in \{1, 2, \dots, n\}$. Moreover, we classify which circulant tournaments $\vec{C}_{2n+1}\langle k \rangle$ are vertex-critical r -dichromatic for every $k \in \{1, 2, \dots, n\}$ and $r \in \{2, 3, 4\}$. Some previous results by Neumann-Lara are generalized.

Keywords: tournament, dichromatic number, vertex-critical r -dichromatic tournament.

2010 Mathematics Subject Classification: 05C20, 05C38.

REFERENCES

- [1] G. Araujo-Pardo and M. Olsen, *A conjecture of Neumann-Lara on infinite families of r -dichromatic circulant tournaments*, Discrete Math. **310** (2010) 489–492. doi:10.1016/j.disc.2009.03.028
- [2] J. Bang-Jensen and G. Gutin, *Digraphs. Theory, Algorithms and Applications*, Second Edition (Springer Monographs in Mathematics, Springer-Verlag London, London, 2009).

- [3] P. Erdős, *Problems and results in number theory and graph theory*, Proceedings of the Ninth Manitoba Conference on Numerical Mathematics and Computing (Univ. Manitoba, Winnipeg, Man., 1979), Congr. Numer. **XXVII** (1979) 3–21.
- [4] P. Erdős, J. Gimbel and D. Kratsch, *Some extremal results in cochromatic and dichromatic theory*, J. Graph Theory **15** (1991) 579–585.
doi:10.1002/jgt.3190150604
- [5] A. Harutyunyan, Brooks-type results for coloring of digraphs, PhD thesis supervised by B. Mohar (Simon Fraser University, 2011).
<http://www.math.univ-toulouse.fr/~aharutyu/thes-short.pdf>
- [6] H. Jacob and H. Meyniel, *Extensions of Turan’s and Brooks theorem and new notions of stability and colouring in digraphs*, Ann. Discrete Math. **17** (1983) 365–370.
- [7] B. Llano and M. Olsen, *On a conjecture of Víctor Neumann-Lara*, Electron. Notes Discrete Math. **30** (2008) 207–212.
doi:10.1016/j.endm.2008.01.036
- [8] B. McKay, Combinatorial Data, published online.
<http://cs.anu.edu.au/~bdm/data>
- [9] V. Neumann-Lara, *The dichromatic number of a digraph*, J. Combin. Theory, Ser. B **33** (1982) 265–270.
doi:10.1016/0095-8956(82)90046-6
- [10] V. Neumann-Lara, *The 3 and 4-dichromatic tournaments of minimum order*, Discrete Math. **135** (1994) 233–243.
doi:10.1016/0012-365X(93)E0113-I
- [11] V. Neumann-Lara, *Vertex critical 4-dichromatic circulant tournaments*, Discrete Math. **170** (1997) 289–291.
doi:10.1016/S0012-365X(96)00128-8
- [12] V. Neumann-Lara, *Dichromatic number, circulant tournaments and Zykov sums of digraphs*, Discuss. Math. Graph Theory **20** (2000) 197–207.
doi:10.7151/dmgt.1119
- [13] V. Neumann-Lara and J. Urrutia, *Vertex critical r -dichromatic tournaments*, Discrete Math. **49** (1984) 83–87.
doi:10.1016/0012-365X(84)90154-7

Received 30 June 2016

Revised 7 April 2016

Accepted 7 April 2016