

## THE LIST DISTINGUISHING NUMBER EQUALS THE DISTINGUISHING NUMBER FOR INTERVAL GRAPHS

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### Abstract

A *distinguishing coloring* of a graph  $G$  is a coloring of the vertices so that every nontrivial automorphism of  $G$  maps some vertex to a vertex with a different color. The *distinguishing number* of  $G$  is the minimum  $k$  such that  $G$  has a distinguishing coloring where each vertex is assigned a color from  $\{1, \dots, k\}$ . A *list assignment* to  $G$  is an assignment  $L = \{L(v)\}_{v \in V(G)}$  of lists of colors to the vertices of  $G$ . A *distinguishing  $L$ -coloring* of  $G$  is a distinguishing coloring of  $G$  where the color of each vertex  $v$  comes from  $L(v)$ . The *list distinguishing number* of  $G$  is the minimum  $k$  such that every list assignment to  $G$  in which  $|L(v)| = k$  for all  $v \in V(G)$  yields a distinguishing  $L$ -coloring of  $G$ . We prove that if  $G$  is an interval graph, then its distinguishing number and list distinguishing number are equal.

**Keywords:** distinguishing, distinguishing number, list distinguishing, interval graph.

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