

## DISTINGUISHING CARTESIAN PRODUCTS OF COUNTABLE GRAPHS

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### Abstract

The distinguishing number  $D(G)$  of a graph  $G$  is the minimum number of colors needed to color the vertices of  $G$  such that the coloring is preserved only by the trivial automorphism. In this paper we improve results about the distinguishing number of Cartesian products of finite and infinite graphs by removing restrictions to prime or relatively prime factors.

**Keywords:** vertex coloring, distinguishing number, automorphisms, infinite graphs, Cartesian and weak Cartesian product.

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