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ALTERNATING-PANCYCLISM IN 2-EDGE-COLORED GRAPHS[1]

Narda Cordero-Michel

and

Hortensia Galeana-Sánchez

Instituto de Matemáticas,
Universidad Nacional Autónoma de México
Ciudad Universitaria, México, D.F., C.P. 04510, México

e-mail: narda@matem.unam.mx
hgaleana@matem.unam.mx

Abstract

An *alternating cycle* in a 2-edge-colored graph is a cycle such that any two consecutive edges have different colors. Let $G_1, \ldots, G_k$ be a collection of pairwise vertex disjoint 2-edge-colored graphs. The *colored generalized sum* of $G_1, \ldots, G_k$, denoted by $\oplus_{i=1}^{k} G_i$, is the set of all 2-edge-colored graphs G such that: (i) $V(G) = \bigcup_{i=1}^{k} V(G_i)$, (ii) $G\langle V(G_i)\rangle \cong G_i$ for $i = 1, \ldots, k$ where $G\langle V(G_i)\rangle$ has the same coloring as G_i and (iii) between each pair of vertices in different summands of G there is exactly one edge, with an arbitrary but fixed color. A graph G in $\oplus_{i=1}^{k} G_i$ will be called a *colored generalized sum* (c.g.s.) and we will say that $e \in E(G)$ is an *exterior edge* if and only if $e \in E(G) \setminus \left(\bigcup_{i=1}^{k} E(G_i)\right)$. The set of exterior edges will be denoted by $E_\oplus$. A 2-edge-colored graph G of order $2n$ is said to be an *alternating-pancyclic graph*, whenever for each $l \in \{2, \ldots, n\}$, there exists an alternating cycle of length $2l$ in G.

The topics of pancyclism and vertex-pancyclism are deeply and widely studied by several authors. The existence of alternating cycles in 2-edge-colored graphs has been studied because of its many applications. In this paper, we give sufficient conditions for a graph $G \in \oplus_{i=1}^{k} G_i$ to be an alternating-pancyclic graph.

Keywords: 2-edge-colored graph, alternating cycle, alternating-pancyclic graph.

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References

[1] J. Bang-Jensen and G. Gutin, *Alternating cycles and paths in edge-coloured multigraphs: A survey*, Discrete Math. **165/166** (1997) 39–60.
doi:10.1016/S0012-365X(96)00160-4

[2] J. Bang-Jensen and G. Gutin, *Alternating cycles and trails in* 2*-edge-coloured complete multigraphs*, Discrete Math. **188** (1998) 61–72.
doi:10.1016/S0012-365X(97)00274-4

[3] J. Bang-Jensen and G.Z. Gutin, Digraphs: Theory, Algorithms and Applications (Springer-Verlag London, Ltd, London, 2009).
doi:10.1007/978-1-84800-998-1

[4] W.S. Chou, Y. Manoussakis, O. Megalakaki, M. Spyratos and Zs. Tuza, *Paths through fixed vertices in edge-colored graphs*, Math. Sci. Hum. **127** (1994) 49–58.

[5] A. Contreras-Balbuena, H. Galeana-Sánchez and I.A. Goldfeder, *A new sufficient condition for the existence of alternating Hamiltonian cycles in* 2*-edge-colored multigraphs*, Discrete Appl. Math. **229** (2017) 55–63.
doi:10.1016/j.dam.2017.04.033

[6] N. Cordero-Michel and H. Galeana-Sánchez, *Vertex alternating-pancyclism in* 2*-edge-colored generalized sum of graphs*, Discrete Appl. Math **284** (2020) 281–289.
doi:10.1016/j.dam.2020.03.047

[7] P. Das, *Pan-alternating cyclic edge-partitioned graphs*, Ars Combin. **14** (1982) 105–114.

[8] D. Dorninger, *On permutations of chromosomes*, in: Contributions to General Algebra 5, Proceedings of the Salzburg Conference, May 29-June 1, 1986, Hölder-Pichler-Tempsky (Ed(s)), (Teubner-Verlag, Stuttgart, 1987) 95–103.

[9] D. Dorninger, *Hamiltonian circuits determining the order of chromosomes*, Discrete Appl. Math. **50** (1994) 159–168.
doi:10.1016/0166-218X(92)00171-H

[10] D. Dorninger and W. Timischl, *Geometrical constraints on Bennett's predictions of chromosome order*, Heredity **59** (1987) 321–325.
doi:10.1038/hdy.1987.138

[11] S. Fujita and C. Magnant, *Properly colored paths and cycles*, Discrete Appl. Math. **159** (2011) 1391–1397.
doi:10.1016/j.dam.2011.06.005

[12] A. Gorbenko and V. Popov, *The Hamiltonian alternating path problem*, IAENG Int. J. Appl. Math. **42** (2012) 204–213.

[13] L. Gourves, A. Lyra, C. Martinhon and J. Monnot, *The minimum reload $s-t$ path, trail and walk problems*, Discrete Appl. Math. **158** (2010) 1404–1417.
doi:10.1016/j.dam.2010.03.009

[14] R. Häggkvist and Y. Manoussakis, *Cycles and paths in bipartite tournaments with spanning configurations*, Combinatorica **9** (1989) 33–38.
doi:10.1007/BF02122681

[15] J. Petersen, *Die Theorie der regulären graphs*, Acta Math. **15** (1891) 193–220. doi:10.1007/BF02392606

[16] P.A. Pevzner, *DNA physical mapping and alternating Eulerian cycles in colored graphs*, Algorithmica **13** (1995) 77–105.
doi:10.1007/BF01188582

[17] G. Wang and H. Li, *Color degree and alternating cycles in edge-colored graphs*, Discrete Math. **309** (2009) 4349–4354.
doi:10.1016/j.disc.2009.01.016

[18] H.C. Wirth and J. Steffan, *Reload cost problems: minimum diameter spanning tree*, Discrete Appl. Math. **113** (2001) 73–85.
doi:10.1016/S0166-218X(00)00392-9

[19] H. Xu, D.M. Kilgour, K.W. Hipel and G. Kemkes, *Using matrices to link conflict evolution and resolution in a graph model*, European J. Oper. Res. **207** (2010) 318–329.
doi:10.1016/j.ejor.2010.03.025

[20] H. Xu, K.W. Li, K.W. Hipel and D.M. Kilgour, *A matrix approach to status quo analysis in the graph model for conflict resolution*, Appl. Math. Comput. **212** (2009) 470–480.
doi:10.1016/j.amc.2009.02.051

[21] H. Xu, K.W. Li, D.M. Kilgour and K.W. Hipel, *A matrix-based approach to searching colored paths in a weighted colored multidigraph*, Appl. Math. Comput. **215** (2009) 353–366.
doi:10.1016/j.amc.2009.04.086

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