

GRAPH OPERATIONS AND NEIGHBORHOOD POLYNOMIALS

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Abstract

The neighborhood polynomial of graph G is the generating function for the number of vertex subsets of G of which the vertices have a common neighbor in G . In this paper, we investigate the behavior of this polynomial under several graph operations. Specifically, we provide an explicit formula for the neighborhood polynomial of the graph obtained from a given graph G by vertex attachment. We use this result to propose a recursive algorithm for the calculation of the neighborhood polynomial. Finally, we prove that the neighborhood polynomial can be found in polynomial-time in the class of k -degenerate graphs.

Keywords: neighborhood complex, neighborhood polynomial, domination polynomial, graph operations, graph degeneracy.

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