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GRAPH CLASSES GENERATED BY MYCIELSKIANS

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Abstract

In this paper we use the classical notion of weak Mycielskian $M'(G)$ of a graph G and the following sequence: $M'_0(G) = G$, $M'_1(G) = M'(G)$, and $M'_n(G) = M'\big(M'_{n-1}(G)\big)$, to show that if G is a complete graph of order p, then the above sequence is a generator of the class of p-colorable graphs. Similarly, using Mycielskian $M(G)$ we show that analogously defined sequence is a generator of the class consisting of graphs for which the chromatic number of the subgraph induced by all vertices that belong to at least one triangle is at most p. We also address the problem of characterizing the latter class in terms of forbidden graphs.

Keywords: Mycielski graphs, graph coloring, chromatic number.

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