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DOMINATION NUMBER OF GRAPHS WITH MINIMUM DEGREE FIVE

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Abstract

We prove that for every graph G on n vertices and with minimum degree five, the domination number $\gamma(G)$ cannot exceed $n/3$. The proof combines an algorithmic approach and the discharging method. Using the same technique, we provide a shorter proof for the known upper bound $4n/11$ on the domination number of graphs of minimum degree four.

Keywords: dominating set, domination number, discharging method.

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