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INDEPENDENCE NUMBER AND PACKING COLORING OF GENERALIZED MYCIELSKI GRAPHS

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Abstract

For a positive integer $k \geqslant 1$, a graph G with vertex set V is said to be k-packing colorable if there exists a mapping $f : V \mapsto \{1, 2, \ldots, k\}$ such that any two distinct vertices x and y with the same color $f(x) = f(y)$ are at distance at least $f(x) + 1$. The packing chromatic number of a graph G, denoted by $\chi_\rho(G)$, is the smallest integer k such that G is k-packing colorable.

In this work, we study both independence and packing colorings in the m-generalized Mycielskian of a graph G, denoted $\mu_m(G)$. We first give an explicit formula for $\alpha(\mu_m(G))$ when m is odd and bounds when m is even. We then use these results to give exact values of $\alpha(\mu_m(K_n))$ for any m and n. Next, we give bounds on the packing chromatic number, χ_ρ, of $\mu_m(G)$. We also prove the existence of large planar graphs whose packing chromatic number is 4. The rest of the paper is focused on packing chromatic numbers of the Mycielskian of paths and cycles.

Keywords: independence number, packing chromatic number, Mycielskians, generalized Mycielskians.

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