Discussiones Mathematicae
Graph Theory 41 (2021) 827–847
doi:10.7151/dmgt.2328

MINIMAL GRAPHS WITH DISJOINT DOMINATING AND PAIRED-DOMINATING SETS

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Abstract

A subset $D \subseteq V_G$ is a dominating set of G if every vertex in $V_G - D$ has a neighbor in D, while D is a paired-dominating set of G if D is a dominating set and the subgraph induced by D contains a perfect matching. A graph G is a *DPDP*-graph if it has a pair (D, P) of disjoint sets of vertices of G such that D is a dominating set and P is a paired-dominating set of G. The study of the *DPDP*-graphs was initiated by Southey and Henning [Cent. Eur. J. Math. 8 (2010) 459–467; J. Comb. Optim. 22 (2011) 217–234]. In this paper, we provide conditions which ensure that a graph is a *DPDP*-graph. In particular, we characterize the minimal *DPDP*-graphs.

Keywords: domination, paired-domination.

2010 Mathematics Subject Classification: 05C69, 05C85.

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Received 20 November 2019
Revised 27 April 2020
Accepted 27 April 2020