

ACHROMATIC NUMBERS FOR CIRCULANT GRAPHS AND DIGRAPHS

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Abstract

In this paper, we determine the achromatic and diachromatic numbers of some circulant graphs and digraphs each one with two lengths and give bounds for other circulant graphs and digraphs with two lengths. In particular, for the achromatic number we state that $\alpha(C_{16q^2+20q+7}(1, 2)) = 8q + 5$, and for the diachromatic number we state that $\text{dac}(\vec{C}_{32q^2+24q+5}(1, 2)) = 8q + 3$. In general, we give the lower bounds $\alpha(C_{4q^2+aq+1}(1, a)) \geq 4q + 1$ and $\text{dac}(\vec{C}_{8q^2+2(a+4)q+a+3}(1, a)) \geq 4q + 3$ when a is a non quadratic residue of $\mathbb{Z}_{4q+1}$ for graphs and $\mathbb{Z}_{4q+3}$ for digraphs, and the equality is attained, in both cases, for $a = 3$.

Finally, we determine the achromatic index for circulant graphs of $q^2 + q + 1$ vertices when the projective cyclic plane of odd order q exists.

Keywords: circulant graphs, complete colorings, achromatic number, achromatic index.

2010 Mathematics Subject Classification: 05C15.

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Received 18 November 2019

Revised 20 April 2020

Accepted 20 April 2020