Discussiones Mathematicae
Graph Theory 41 (2021) 713–724
doi:10.7151/dmgt.2327

ACHROMATIC NUMBERS FOR CIRCULANT GRAPHS AND DIGRAPHS

Gabriela Araujo-Pardo
Juan José Montellano-Ballesteros

Instituto de Matemáticas
Universidad Nacional Autónoma de México, México

e-mail: garaujo@matem.unam.mx
juancho@matem.unam.mx

Mika Olsen

Departamento de Matemáticas Aplicadas y Sistemas
UAM-Cuajimalpa, México

e-mail: olsen@correo.cua.uam.mx

and

Christian Rubio-Montiel

División de Matemáticas e Ingeniería, FES Acatlán
Universidad Nacional Autónoma de México, Naucalpan, Mexico

e-mail: christian.rubio@acatlan.unam.mx

Abstract

In this paper, we determine the achromatic and diachromatic numbers of some circulant graphs and digraphs each one with two lengths and give bounds for other circulant graphs and digraphs with two lengths. In particular, for the achromatic number we state that $\alpha(C_{16q^2+20q+7}(1,2)) = 8q+5$, and for the diachromatic number we state that $dac(\overrightarrow{C}_{32q^2+24q+5}(1,2)) = 8q+3$. In general, we give the lower bounds $\alpha(C_{4q^2+aq+1}(1,a)) \geq 4q+1$ and $dac(\overrightarrow{C}_{8q^2+2(a+4)q+a+3}(1,a)) \geq 4q+3$ when a is a non quadratic residue of $\mathbb{Z}_{4q+1}$ for graphs and $\mathbb{Z}_{4q+3}$ for digraphs, and the equality is attained, in both cases, for $a = 3$.

Finally, we determine the achromatic index for circulant graphs of $q^2 + q + 1$ vertices when the projective cyclic plane of odd order q exists.

Keywords: circulant graphs, complete colorings, achromatic number, achromatic index.

2010 Mathematics Subject Classification: 05C15.

References

[1] G. Araujo-Pardo, J.J. Montellano-Ballesteros, M. Olsen and C. Rubio-Montiel, *The diachromatic number of digraphs*, Electron. J. Combin. **25** (2018) #P3.51 doi:10.37236/7807

[2] G. Araujo-Pardo, J.J. Montellano-Ballesteros, C. Rubio-Montiel and R. Strausz, *On the pseudoachromatic index of the complete graph* III, Graphs Combin. **34** (2018) 277–287. doi:10.1007/s00373-017-1872-6

[3] G. Araujo-Pardo, J. J. Montellano-Ballesteros and R. Strausz, *On the pseudoachromatic index of the complete graph*, J. Graph Theory **66** (2011) 89–97. doi:10.1002/jgt.20491

[4] G. Araujo-Pardo, J.J. Montellano-Ballesteros, R. Strausz and C. Rubio-Montiel, *On the pseudoachromatic index of the complete graph* II, Bol. Soc. Mat. Mex. **20** (2014) 17–28. doi:10.1007/s40590-014-0007-9

[5] G. Araujo-Pardo and C. Rubio-Montiel, *On $\omega\psi$-perfect graphs*, Ars Combin. **141** (2018) 375–387.

[6] F. Bories and J. Jolivet, *On complete colorings of graphs*, in: Recent Advances in Graph Theory, Proc. Second Czechoslovak Sympos., Prague, 1974, (Academia, Prague, 1975) 75–87.

[7] A. Bouchet, *Indice achromatique des graphes multiparti complets et réguliers*, Cahiers Centre Études Rech. Opér. **20** (1978) 331–340.

[8] D.M. Burton, Elementary Number Theory, Second Edition (W.C. Brown Publishers, Dubuque, IA, 1989).

[9] N. Cairnie and K. Edwards, *Some results on the achromatic number*, J. Graph Theory **26** (1997) 129–136. doi:10.1002/(SICI)1097-0118(199711)26:3⟨129::AID-JGT3⟩3.0.CO;2-T

[10] G. Chartrand and P. Zhang, Chromatic Graph Theory (Chapman and Hall/CRC Press, New York, 2008). doi:10.1201/9781584888017

[11] M. Dębski, Z. Lonc and P. Rzążewski, *Achromatic and harmonious colorings of circulant graphs*, J. Graph Theory **87** (2018) 18–34. doi:10.1002/jgt.22137

[12] K.J. Edwards, *Harmonious chromatic number of directed graphs*, Discrete Appl. Math. **161** (2013) 369–376. doi:10.1016/j.dam.2012.09.003

[13] F. Harary, S. Hedetniemi and G. Prins, *An interpolation theorem for graphical homomorphisms*, Port. Math. **26** (1967) 453–462.

[14] P. Hell and D.J. Miller, *Graph with given achromatic number*, Discrete Math. **16** (1976) 195–207. doi:10.1016/0012-365X(76)90099-6

[15] M. Horňák, *Achromatic index of K_{12}*, Ars Combin. **45** (1997) 271–275.

[16] M. Horňák, Š. Pčola and M. Woźniak, *On the achromatic index of K_{q^2+q} for a prime q*, Graphs Combin. **20** (2004) 191–203.
doi:10.1007/s00373-004-0550-7

[17] R.E. Jamison, *On the edge achromatic numbers of complete graphs*, Discrete Math. **74** (1989) 99–115.
doi:10.1016/0012-365X(89)90202-1

[18] R.E. Jamison, *On the achromatic index of K_{12}*, in: Proceedings of the Twenty-second Southeastern Conference on Combinatorics, Graph Theory and Computing, Baton Rouge, 1991, Congr. Numer. **81** (1991) 143–148.

[19] F. Kárteszi, Introduction to Finite Geometries (North-Holland Publ. Co., New York, 1976).

[20] V. Neumann-Lara, *The dichromatic number of a digraph*, J. Combin. Theory Ser. B **33** (1982) 265–270.
doi:10.1016/0095-8956(82)90046-6

[21] É. Sopena, *Complete oriented colourings and the oriented achromatic number*, Discrete Appl. Math. **173** (2014) 102–112.
doi:10.1016/j.dam.2014.03.015

[22] C. M. Turner, R. Rowley, R. Jamison and R. Laskar, *The edge achromatic number of small complete graphs*, Congr. Numer. **62** (1988) 21–36.

[23] V. Yegnanarayanan, *The pseudoachromatic number of a graph*, Southeast Asian Bull. Math. **24** (2000) 129–136.
doi:10.1007/s10012-000-0129-z

[24] V. Yegnanarayanan, *Graph colourings and partitions*, Theoret. Comput. Sci. **263** (2001) 59–74.
doi:10.1016/S0304-3975(00)00231-0

Received 18 November 2019
Revised 20 April 2020
Accepted 20 April 2020