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PROPER RAINBOW CONNECTION NUMBER OF GRAPHS

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Abstract

A path in an edge-coloured graph is called a *rainbow path* if its edges receive pairwise distinct colours. An edge-coloured graph is said to be *rainbow connected* if any two distinct vertices of the graph are connected by a rainbow path. The minimum k for which there exists such an edge-colouring is the rainbow connection number $rc(G)$ of G. Recently, Bau *et al.* [*Rainbow connectivity in some Cayley graphs*, Australas. J. Combin. 71 (2018) 381–393] introduced this concept with the additional requirement that the edge-colouring must be proper. The *proper rainbow connection number* of G, denoted by $prc(G)$, is the minimum number of colours needed in order to make it properly rainbow connected. Obviously, $prc(G) \geq \max\{rc(G), \chi'(G)\}$.

In this paper we first prove an improved upper bound $prc(G) \leq n$ for every connected graph G of order $n \geq 3$. Next we show that the difference $prc(G) - \max\{rc(G), \chi'(G)\}$ can be arbitrarily large. Finally, we present several sufficient conditions for graph classes satisfying $prc(G) = \chi'(G)$.

Keywords: edge-colouring, rainbow connection number, proper rainbow connection number.

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