

ON ANTIPODAL AND DIAMETRICAL PARTIAL CUBES

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Abstract

We prove that any diametrical partial cube of diameter at most 6 is antipodal. Because any antipodal graph is harmonic, this gives a partial answer to a question of Fukuda and Handa [*Antipodal graphs and oriented matroids*, Discrete Math. 111 (1993) 245–256] whether any diametrical partial cube is harmonic, and improves a previous result of Klavžar and Kovše [*On even and harmonic-even partial cubes*, Ars Combin. 93 (2009) 77–86].

Keywords: diametrical graph, harmonic graph, antipodal graph, partial cube, diameter, isometric dimension.

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REFERENCES

- [1] K. Balakrishnan, B. Brešar, M. Changat, S. Klavžar, I. Peterin and A.R. Subhamathi, *Almost self-centered median and chordal graphs*, Taiwanese J. Math. **16** (2012) 1911–1922.
<https://doi.org/10.11650/twjm/1500406804>
- [2] V. Chepoi, *Isometric subgraphs of Hamming graphs and d-convexity*, Cybernet. Systems Anal. **24** (1988) 6–11.
<https://doi.org/10.1007/BF01069520>
- [3] J. Desharnais, *Maille et Plongements de Graphes Antipodaux* (Mémoire de Maitrise, Université de Montréal, 1993).
- [4] D.Ž. Djoković, *Distance-preserving subgraphs of hypercubes*, J. Combin. Theory Ser. B **14** (1973) 263–267.
[https://doi.org/10.1016/0095-8956\(73\)90010-5](https://doi.org/10.1016/0095-8956(73)90010-5)
- [5] V.V. Firsov, *Isometric embedding of a graph in a Boolean cube*, Cybernet. System Anal. **1** (1965) 112–113.
<https://doi.org/10.1007/BF01074705>

- [6] K. Fukuda and K. Handa, *Antipodal graphs and oriented matroids*, Discrete Math. **111** (1993) 245–256.
[https://doi.org/10.1016/0012-365X\(93\)90159-Q](https://doi.org/10.1016/0012-365X(93)90159-Q)
- [7] F. Glivjak, A. Kotzig and J. Plesnik, *Remarks on graphs with a central symmetry*, Monatsh. Math. **74** (1970) 302–307.
<https://doi.org/10.1007/BF01302697>
- [8] F. Göbel and H.J. Veldman, *Even graphs*, J. Graph Theory **10** (1986) 225–239.
<https://doi.org/10.1002/jgt.3190100212>
- [9] R. Hammack, W. Imrich and S. Klavžar, *Handbook of Product Graphs*, Second Edition (CRC Press, Boca Raton, 2011).
<https://doi.org/10.1201/b10959>
- [10] K. Handa, *Bipartite graphs with balanced (a, b) -partitions*, Ars Combin. **51** (1999) 113–119.
- [11] S. Klavžar and M. Kovše, *On even and harmonic-even partial cubes*, Ars Combin. **93** (2009) 77–86.
- [12] K. Knauer and T. Marc, *On tope graphs of complexes of oriented matroids* (2017).
arXiv:1701.05525
- [13] A. Kotzig, *Centrally symmetric graphs*, Czechoslovak Math. J. **18** (1968) 605–615, in Russian.
- [14] A. Kotzig and P. Laufer, *Generalized S -graphs*, preprint CRM-779, Centre de Recherches Mathématiques (Université de Montréal, 1978).
- [15] T. Marc, *There are no finite partial cubes of girth more than 6 and minimum degree at least 3*, European J. Combin. **55** (2016) 62–72.
<https://doi.org/10.1016/j.ejc.2016.01.005>
- [16] H.M. Mulder, *n -cubes and median graphs*, J. Graph Theory **4** (1980) 107–110.
<https://doi.org/10.1002/jgt.3190040112>
- [17] K.R. Parthasarathy and R. Nandakumar, *Unique eccentric point graphs*, Discrete Math. **46** (1983) 69–74.
[https://doi.org/10.1016/0012-365X\(83\)90271-6](https://doi.org/10.1016/0012-365X(83)90271-6)
- [18] N. Polat, *On some characterizations of antipodal partial cubes*, Discuss. Math. Graph Theory **39** (2019) 439–453.
<https://doi.org/10.7151/dmgt.2083>
- [19] G. Sabidussi, *Graphs without dead ends*, European J. Combin. **17** (1996) 69–87.
<https://doi.org/10.1006/eujc.1996.0006>
- [20] P.M. Winkler, *Isometric embeddings in products of complete graphs*, Discrete Appl. Math. **7** (1984) 221–225.
[https://doi.org/10.1016/0166-218X\(84\)90069-6](https://doi.org/10.1016/0166-218X(84)90069-6)

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