Discussiones Mathematicae
Graph Theory 41 (2021) 1127–1145
https://doi.org/10.7151/dmgt.2305

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DMGT Page

ON ANTIPODAL AND DIAMETRICAL PARTIAL CUBES

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Abstract

We prove that any diametrical partial cube of diameter at most 6 is antipodal. Because any antipodal graph is harmonic, this gives a partial answer to a question of Fukuda and Handa [*Antipodal graphs and oriented matroids*, Discrete Math. 111 (1993) 245–256] whether any diametrical partial cube is harmonic, and improves a previous result of Klavžar and Kovše [*On even and harmonic-even partial cubes*, Ars Combin. 93 (2009) 77–86].

Keywords: diametrical graph, harmonic graph, antipodal graph, partial cube, diameter, isometric dimension.

2010 Mathematics Subject Classification: 05C12, 05C75.

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Received 17 November 2018
Accepted 24 August 2019