

A NEW FRAMEWORK TO APPROACH VIZING'S CONJECTURE

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Abstract

We introduce a new setting for dealing with the problem of the domination number of the Cartesian product of graphs related to Vizing's conjecture. The new framework unifies two different approaches to the conjecture. The most common approach restricts one of the factors of the product to some class of graphs and proves the inequality of the conjecture then holds when the other factor is any graph. The other approach utilizes the so-called Clark-Suen partition for proving a weaker inequality that holds for all pairs of graphs. We demonstrate the strength of our framework by improving the bound of Clark and Suen as follows: $\gamma(X \square Y) \geq \max \left\{ \frac{1}{2} \gamma(X) \gamma_t(Y), \frac{1}{2} \gamma_t(X) \gamma(Y) \right\}$, where γ stands for the domination number, γ_t is the total domination number, and $X \square Y$ is the Cartesian product of graphs X and Y .

Keywords: Cartesian product, total domination, Vizing's conjecture, Clark and Suen bound.

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