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A NEW FRAMEWORK TO APPROACH VIZING'S CONJECTURE

Boštjan Brešar[a,b], Bert L. Hartnell[c], Michael A. Henning[d]

Kirsti Kuenzel[e] and Douglas F. Rall[f]

[a] *Faculty of Natural Sciences and Mathematics*
University of Maribor, Slovenia

[b] *Institute of Mathematics, Physics and Mechanics*
Ljubljana, Slovenia

[c] *Saint Mary's University, Halifax, Nova Scotia, Canada*

[d] *Department of Mathematics and Applied Mathematics*
University of Johannesburg, Johannesburg, South Africa

[e] *Department of Mathematics, Western New England University*
Springfield, MA, USA

[f] *Department of Mathematics, Furman University, Greenville, SC, USA*

Abstract

We introduce a new setting for dealing with the problem of the domination number of the Cartesian product of graphs related to Vizing's conjecture. The new framework unifies two different approaches to the conjecture. The most common approach restricts one of the factors of the product to some class of graphs and proves the inequality of the conjecture then holds when the other factor is any graph. The other approach utilizes the so-called Clark-Suen partition for proving a weaker inequality that holds for all pairs of graphs. We demonstrate the strength of our framework by improving the bound of Clark and Suen as follows: $\gamma(X \square Y) \geq \max\left\{\frac{1}{2}\gamma(X)\gamma_t(Y), \frac{1}{2}\gamma_t(X)\gamma(Y)\right\}$, where γ stands for the domination number, γ_t is the total domination number, and $X \square Y$ is the Cartesian product of graphs X and Y.

Keywords: Cartesian product, total domination, Vizing's conjecture, Clark and Suen bound.

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