# ON LIST EQUITABLE TOTAL COLORINGS OF THE GENERALIZED THETA GRAPH 

Jeffrey A. Mudrock, Max Marsh

AND
Tim Wagstrom
Department of Mathematics
College of Lake County
Grayslake, IL 60030
e-mail: jumudrock@clcillinois.edu


#### Abstract

In 2003, Kostochka, Pelsmajer, and West introduced a list analogue of equitable coloring called equitable choosability. A $k$-assignment, $L$, for a graph $G$ assigns a list, $L(v)$, of $k$ available colors to each $v \in V(G)$, and an equitable $L$-coloring of $G$ is a proper coloring, $f$, of $G$ such that $f(v) \in L(v)$ for each $v \in V(G)$ and each color class of $f$ has size at most $\lceil|V(G)| / k\rceil$. Graph $G$ is equitably $k$-choosable if $G$ is equitably $L$-colorable whenever $L$ is a $k$-assignment for $G$. In 2018, Kaul, Mudrock, and Pelsmajer subsequently introduced the List Equitable Total Coloring Conjecture which states that if $T$ is a total graph of some simple graph, then $T$ is equitably $k$-choosable for each $k \geq \max \left\{\chi_{\ell}(T), \Delta(T) / 2+2\right\}$ where $\Delta(T)$ is the maximum degree of a vertex in $T$ and $\chi_{\ell}(T)$ is the list chromatic number of $T$. In this paper, we verify the List Equitable Total Coloring Conjecture for subdivisions of stars and the generalized theta graph.


Keywords: graph coloring, total coloring, equitable coloring, list coloring, equitable choosability.
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