

ON LIST EQUITABLE TOTAL COLORINGS OF THE GENERALIZED THETA GRAPH

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Abstract

In 2003, Kostochka, Pelsmayer, and West introduced a list analogue of equitable coloring called equitable choosability. A k -assignment, L , for a graph G assigns a list, $L(v)$, of k available colors to each $v \in V(G)$, and an equitable L -coloring of G is a proper coloring, f , of G such that $f(v) \in L(v)$ for each $v \in V(G)$ and each color class of f has size at most $\lceil |V(G)|/k \rceil$. Graph G is equitably k -choosable if G is equitably L -colorable whenever L is a k -assignment for G . In 2018, Kaul, Mudrock, and Pelsmayer subsequently introduced the List Equitable Total Coloring Conjecture which states that if T is a total graph of some simple graph, then T is equitably k -choosable for each $k \geq \max\{\chi_\ell(T), \Delta(T)/2 + 2\}$ where $\Delta(T)$ is the maximum degree of a vertex in T and $\chi_\ell(T)$ is the list chromatic number of T . In this paper, we verify the List Equitable Total Coloring Conjecture for subdivisions of stars and the generalized theta graph.

Keywords: graph coloring, total coloring, equitable coloring, list coloring, equitable choosability.

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REFERENCES

- [1] M. Behzad, *Graphs and their Chromatic Numbers*, Ph.D. Thesis (Michigan State University, 1965).
- [2] J. Beier, J. Fierson, R. Haas, H.M. Russel and K. Shavo, *Classifying coloring graphs*, *Discrete Math.* **339** (2016) 2100–2112.
<https://doi.org/10.1016/j.disc.2016.03.003>

- [3] J.I. Brown, C.A. Hickman, A.D. Sokal and D.G. Wagner, *On the chromatic roots of generalized theta graphs*, J. Combin. Theory Ser. B **83** (2001) 272–297.
<https://doi.org/10.1006/jctb.2001.2057>
- [4] J.M. Carraher, T. Mahoney, G.J. Puelo and D.B. West, *Sum-paintability of generalized theta-graphs*, Graphs Combin. **31** (2015) 1325–1334.
<https://doi.org/10.1007/s00373-014-1441-1>
- [5] B.-L. Chen, K.-W. Lih and P.-L. Wu, *Equitable coloring and the maximum degree*, European J. Combin. **15** (1994) 443–447.
<https://doi.org/10.1006/eujc.1994.1047>
- [6] T. Chunling, L. Xiaohui, Y. Yuansheng and L. Zhihe, *Equitable total coloring of $C_m \square C_n$* , Discrete Appl. Math. **157** (2009) 596–601.
<https://doi.org/10.1016/j.dam.2008.08.030>
- [7] A. Dong and J. Wu, *Equitable coloring and equitable choosability of planar graphs without chordal 4- and 6-cycles* (2018).
arXiv:1806.01064
- [8] A. Dong and X. Zhang, *Equitable coloring and equitable choosability of graphs with small maximum average degree*, Discuss. Math. Graph Theory **38** (2018) 829–839.
<https://doi.org/10.7151/dmgt.2049>
- [9] P. Erdős, *Problem 9*, in: Theory of Graphs and Its Applications, M. Fiedler (Ed(s)), (Proc. Sympos., Smolenice, 1963, Publ. House Czechoslovak Acad. Sci. Prague, 1964) 159.
- [10] P. Erdős, A.L. Rubin and H. Taylor, *Choosability in graphs*, Congr. Numer. **26** (1979) 125–127.
- [11] H.-L. Fu, *Some results on equalized total coloring*, Congr. Numer. **102** (1994) 111–119.
- [12] H. Furmańczyk and R. Zuazua, *Equitable total coloring of corona of cubic graphs*, Discuss. Math. Graph Theory **41** (2021) 1147–1163.
<https://doi.org/10.7151/dmgt.2245>
- [13] M.A. Gang and M.A. Ming, *The equitable chromatic number of some join graphs*, Open J. Appl. Sci. (2012) 96–99.
<https://doi.org/10.4236/ojapps.2012.24B023>
- [14] K. Gong, Z.-F. Zhang and J.-F. Wang, *Equitable total coloring of $F_n \vee W_n$* , Acta Math. Appl. Sin. Engl. Ser. **25** (2009) 83–86.
<https://doi.org/10.1007/s10255-006-6031-4>
- [15] A. Hajnal and E. Szemerédi, *Proof of a conjecture of Erdős*, in: Combinatorial Theory and Its Applications, A. Rényi and V.T. Sós (Ed(s)), (Vol. II, North-Holland, Amsterdam, 1970) 601–623.
- [16] S. Janson and A. Ruciński, *The infamous upper tail*, Random Structures Algorithms **20** (2002) 317–342.
<https://doi.org/10.1002/rsa.10031>

- [17] H. Kaul and S.H. Jacobson, *New global optima results for the Kauffman NK model: handling dependency*, Math. Program. **108** (2006) 475–494.
<https://doi.org/10.1007/s10107-006-0719-3>
- [18] H. Kaul, J.A. Mudrock and M.J. Pelsmayer, *Total equitable list coloring*, Graphs Combin. **34** (2018) 1637–1649.
<https://doi.org/10.1007/s00373-018-1965-x>
- [19] H.A. Kierstead and A.V. Kostochka, *Equitable list coloring of graphs with bounded degree*, J. Graph Theory **74** (2013) 309–334.
<https://doi.org/10.1002/jgt.21710>
- [20] A.V. Kostochka, M.J. Pelsmayer and D.B. West, *A list analogue of equitable coloring*, J. Graph Theory **44** (2003) 166–177.
<https://doi.org/10.1002/jgt.10137>
- [21] D. Laiche, I. Bouchemakh and É. Sopena, *On the packing coloring of undirected and oriented generalized theta graphs*, Australas. J. Combin. **66** (2016) 310–329.
- [22] M.E. Leidner, *A Study of the Total Colorings of Graphs*, Ph.D. Thesis (University of Louisville, 2012).
- [23] Q. Li and Y. Bu, *Equitable list coloring of planar graphs without 4- and 6-cycles*, Discrete Math. **309** (2009) 280–287.
<https://doi.org/10.1016/j.disc.2007.12.070>
- [24] R. Li, H. Broersma and S. Zhang, *Properly edge-colored theta graphs in edge-colored complete graphs*, Graphs Combin. **35** (2019) 261–286.
<https://doi.org/10.1007/s00373-018-1989-2>
- [25] K.-W. Lih, *The equitable coloring of graphs*, in: Handbook of Combinatorial Optimization, D.-Z. Du and P. Pardalos (Ed(s)), (Vol. III, Kluwer, Dordrecht, 1998) 543–566.
- [26] K.-W. Lih and P.-L. Wu, *On equitable coloring of bipartite graphs*, Discrete Math. **151** (1996) 155–160.
[https://doi.org/10.1016/0012-365X\(94\)00092-W](https://doi.org/10.1016/0012-365X(94)00092-W)
- [27] W. Meyer, *Equitable coloring*, Amer. Math. Monthly **80** (1973) 920–922.
<https://doi.org/10.1080/00029890.1973.11993408>
- [28] J. Mudrock, *On the List Coloring Problem and Its Equitable Variants*, Ph.D. Thesis (Illinois Institute of Technology, 2018).
- [29] J. Mudrock, M. Chase, I. Kadera and T. Wagstrom, *A note on the equitable choosability of complete bipartite graphs*, Discuss. Math. Graph Theory **41** 2021 1091–1101.
<https://doi.org/10.7151/dmgt.2232>
- [30] K. Nakprasit, personal communication, 2002.
- [31] S.V. Pemmaraju, *Equitable colorings extend Chernoff-Hoeffding bounds*, Proceedings of the 5th International Workshop on Randomization and Approximation Techniques in Computer Science (2001) 285–296.

- [32] G. Sathiamoorthy and T.N. Janakiraman, *Graceful labeling of generalized theta graphs*, Nat. Acad. Sci. Lett. **41** (2018) 121–122.
<https://doi.org/10.1007/s40009-018-0625-2>
- [33] A. Tucker, *Perfect graphs and an application to optimizing municipal services*, SIAM Rev. **15** (1973) 585–590.
<https://doi.org/10.1137/1015072>
- [34] V.G. Vizing, *Some unsolved problems in graph theory*, Ups. Mat. Nauk. **23** (1968) 117–134, in Russian, English translation in Russian Math. Surveys **23** (1968) 125–141.
<https://doi.org/10.1070/RM1968v023n06ABEH001252>
- [35] V.G. Vizing, *Coloring the vertices of a graph in prescribed colors*, Diskret. Analiz. **29**, Metody Diskret. Anal. v Teorii Kodovi Skhem **101** (1976) 3–10.
- [36] W.-F. Wang, *Equitable total coloring of graphs with maximum degree 3*, Graphs Combin. **18** (2002) 677–685.
<https://doi.org/10.1007/s003730200051>
- [37] D.B. West, *Introduction to Graph Theory* (Prentice Hall, NJ, 2001).
- [38] H.P. Yap and Y. Zhang, *The equitable Δ -coloring conjecture holds for outerplanar graphs*, Bull. Inst. Math. Acad. Sin. **25** (1997) 143–149.
- [39] Z. Zhang, W. Wang, S. Bau and J. Li, *On the equitable total colorings of some join graphs*, J. Inf. Comput. Sci. **2** (2005) 829–834.
- [40] J. Zhu and Y. Bu, *Equitable list coloring of planar graphs without short cycles*, Theoret. Comput. Sci. **407** (2008) 21–28.
<https://doi.org/10.1016/j.tcs.2008.04.018>
- [41] X. Zhang and J.-L. Wu, *On equitable and equitable list colorings of series-parallel graphs*, Discrete Math. **311** (2011) 800–803.
<https://doi.org/10.1016/j.disc.2011.02.001>
- [42] J. Zhu and Y. Bu, *Equitable and equitable list colorings of graphs*, Theoret. Comput. Sci. **411** (2010) 3873–3876.
<https://doi.org/10.1016/j.tcs.2010.06.027>
- [43] J. Zhu, Y. Bu and X. Min, *Equitable list-coloring for C_5 -free plane graphs without adjacent triangles*, Graphs Combin. **31** (2015) 795–804.
<https://doi.org/10.1007/s00373-013-1396-7>

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