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ON LIST EQUITABLE TOTAL COLORINGS OF THE GENERALIZED THETA GRAPH

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Abstract

In 2003, Kostochka, Pelsmajer, and West introduced a list analogue of equitable coloring called equitable choosability. A k-assignment, L, for a graph G assigns a list, $L(v)$, of k available colors to each $v \in V(G)$, and an equitable L-coloring of G is a proper coloring, f, of G such that $f(v) \in L(v)$ for each $v \in V(G)$ and each color class of f has size at most $\lceil |V(G)|/k \rceil$. Graph G is equitably k-choosable if G is equitably L-colorable whenever L is a k-assignment for G. In 2018, Kaul, Mudrock, and Pelsmajer subsequently introduced the List Equitable Total Coloring Conjecture which states that if T is a total graph of some simple graph, then T is equitably k-choosable for each $k \geq \max\{\chi_\ell(T), \Delta(T)/2 + 2\}$ where $\Delta(T)$ is the maximum degree of a vertex in T and $\chi_\ell(T)$ is the list chromatic number of T. In this paper, we verify the List Equitable Total Coloring Conjecture for subdivisions of stars and the generalized theta graph.

Keywords: graph coloring, total coloring, equitable coloring, list coloring, equitable choosability.

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