Discussiones Mathematicae
Graph Theory 40 (2020) 677–690
doi:10.7151/dmgt.2288

Full PDF
DMGT Page

GRAPHS WHOSE A_α-SPECTRAL RADIUS DOES NOT EXCEED 2

Jian Feng Wang[1]

School of Mathematics and Statistics
Shandong University of Technology
Zibo 255049, P.R. China

e-mail: jfwang@sdut.edu.com

Jing Wang, Xiaogang Liu

Department of Applied Mathematics
Northwestern Polytechnical University
Xi'an, 710072, P.R. China

e-mail: jwang66@aliyun.com
xiaogliu@nwpu.edu.cn

AND

Francesco Belardo

Department of Mathematics and Applications "R. Caccioppoli"
University of Naples "Federico II", I-80126 Naples, Italy

e-mail: fbelardo@unina.it

This paper is dedicated to the memory of our excellent colleague Slobodan K. Simić who recently passed away.

Abstract

Let $A(G)$ and $D(G)$ be the adjacency matrix and the degree matrix of a graph G, respectively. For any real $\alpha \in [0,1]$, we consider $A_\alpha(G) = \alpha D(G) + (1-\alpha)A(G)$ as a graph matrix, whose largest eigenvalue is called the A_α-spectral radius of G. We first show that the smallest limit point for the A_α-spectral radius of graphs is 2, and then we characterize the connected graphs whose A_α-spectral radius is at most 2. Finally, we show that all such graphs, with four exceptions, are determined by their A_α-spectra.

Keywords: A_α-matrix, Smith graphs, limit point, spectral radius, index.
2010 Mathematics Subject Classification: 05C50.

[1]Corresponding author.

References

[1] F. Belardo, E.M. Li Marzi, S.K. Simić and J.F. Wang, *Graphs whose signless Laplacian spectral radius does not exceed the Hoffman limit value*, Linear Algebra Appl. **435** (2011) 2913–2920.
doi:10.1016/j.laa.2011.05.006

[2] A.E. Brouwer and A. Neumaier, *The graphs with spectral radius between* 2 *and* $\sqrt{2+\sqrt{5}}$, Linear Algebra Appl. **114/115** (1989) 273–276.
doi:10.1016/0024-3795(89)90466-7

[3] S.M. Cioabă, E.R. van Dam, J.H. Koolen and J.-H. Lee, *Asymptotic results on the spectral radius and the diameter of graphs*, Linear Algebra Appl. **432** (2010) 722–737.
doi:10.1016/j.laa.2009.09.016

[4] D.M. Cvetković, M. Doob and I. Gutman, *On graphs whose spectral radius does not exceed* $(2+\sqrt{5})^{1/2}$, Ars Combin. **14** (1982) 225–239.

[5] E.R. van Dam and W.H. Haemers, *Which graphs are determined by their spectrum*?, Linear Algebra Appl. **373** (2003) 241–272.
doi:10.1016/S0024-3795(03)00483-X

[6] E.R. van Dam and W.H. Haemers, *Developments on spectral characterizations of graphs*, Discrete Math. **309** (2009) 576–586.
doi:10.1016/j.disc.2008.08.019

[7] N. Ghareghani, G.R. Omidi and B. Tayfeh-Rezaie, *Spectral characterization of graphs with index at most* $\sqrt{2+\sqrt{5}}$, Linear Algebra Appl. **420** (2007) 483–489.
doi:10.1016/j.laa.2006.08.009

[8] K. Guo and B. Mohar, *Digraphs with Hermitian spectral radius below* 2 *and their cospectrality with paths*, Discrete Appl. **340** (2017) 2616–2632.
doi:10.1016/j.disc.2017.01.018

[9] A.J. Hoffman, *On limit points on spectral radii of non-negative symmetric integral matrices*, in: Graph Theory and Applications, Y. Alavi, D.R. Lick, A.T. White (Ed(s)), Lecture Notes in Math. **303** (Springer-Verlag, Berlin, 1972) 165–172.

[10] A.J. Hoffman and J.H. Smith, *On the spectral radii of topologically equivalent graphs*, in: Recent Advances in Graph Theory, M. Fiedler (Ed(s)), (Academia Praha, Prague, 1975) 273–281.

[11] H.Q. Lin, X. Huang and J. Xue, *A note on the* A_α*-spectral radius of graphs*, Linear Algebra Appl. **557** (2018) 430–437.
doi:10.1016/j.laa.2018.08.008

[12] H.Q. Lin, X.G. Liu and J. Liu, *Graphs determined by their* A_α*-spectra*, Discrete Math. **342** (2019) 441–450.
doi:10.1016/j.disc.2018.10.006

[13] H.Q. Lin, J. Xue and J.L. Shu, *On the* A_α*-spectra of graphs*, Linear Algebra Appl. **556** (2018) 210–219.
doi:10.1016/j.laa.2018.07.003

[14] X.G. Liu and S.Y. Liu, *On the A_α-characteristic polynomial of a graph*, Linear Algebra Appl. **546** (2018) 274–288.
doi:10.1016/j.laa.2018.02.014

[15] L. Lu and S. Man, *Connected hypergraphs with small spectral radius*, Linear Algebra Appl. **509** (2016) 206–227.
doi:10.1016/j.laa.2016.07.013

[16] V. Nikiforov, *Merging the A-and Q-spectral theories*, Appl. Anal. Discrete Math. **11** (2017) 81–107.
doi:10.2298/AADM1701081N

[17] V. Nikiforov, G. Pastén, O. Rojo and R.L. Soto, *On the A_α-spectra of trees*, Linear Algebra Appl. **520** (2017) 286–305.
doi:10.1016/j.laa.2017.01.029

[18] V. Nikiforov and O. Rojo, *A note on the positive semidefiniteness of $A_\alpha(G)$*, Linear Algebra Appl. **519** (2017) 156–163.
doi:10.1016/j.laa.2016.12.042

[19] J.H. Smith, *Some properties of the spectrum of a graph*, in: Combinatorial Structures and their Applications, R. Guy, H. Hanani, N. Sauer, J. Schonheim (Ed(s)), (Gordon and Breach, New York, 1970) 403–406.

[20] J.F. Wang and F. Belardo, *Spectral characterizations of graphs with small spectral radius*, Linear Algebra Appl. **437** (2012) 2408–2416.
doi:10.1016/j.laa.2012.06.028

[21] J.F. Wang, F. Belardo, Q.X. Huang and E.M. Li Marzi, *On graphs whose Laplacian index does not exceed* 4.5, Linear Algebra Appl. **438** (2013) 1541–1550.
doi:10.1016/j.laa.2011.02.043

[22] J.F. Wang, Q.X. Huang, F. Belardo and E.M. Li Marzi, *On graphs whose signless Laplacian index does not exceed* 4.5, Linear Algebra Appl. **431** (2009) 162–178.
doi:10.1016/j.laa.2009.02.017

[23] J.F. Wang, Q.X. Huang, X. An and F. Belardo, *Some notes on graphs whose spectral radius is close to* $\frac{3}{2}\sqrt{2}$, Linear Algebra Appl. **429** (2008) 1606–1618.
doi:10.1016/j.laa.2008.04.034

[24] R. Woo and A. Neumaier, *On graphs whose spectral radius is bounded by* $\frac{3}{2}\sqrt{2}$, Graphs Combin. **23** (2007) 713–726.
doi:10.1007/s00373-007-0745-9

Received 4 October 2019
Revised 28 November 2019
Accepted 12 December 2019