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TURÁN'S THEOREM IMPLIES STANLEY'S BOUND

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Dedicated to the memory of Slobodan K. Simić

Abstract

Let G be a graph with m edges and let ρ be the largest eigenvalue of its adjacency matrix. It is shown that

$$\rho \le \sqrt{2\left(1-\left\lfloor 1/2+\sqrt{2m+1/4}\right\rfloor^{-1}\right)m},$$

improving the well-known bound of Stanley. Moreover, writing ω for the clique number of G and W_k for the number of its walks on k vertices, it is shown that the sequence

$$\left\{\left((1-1/\omega)\,W_{2^k}\right)^{1/2^k}\right\}_{k=1}^{\infty}$$

is nonincreasing and converges to ρ.

Keywords: graph spectral radius, Stanley's bound, Turán's theorem, clique number, Motzkin-Straus's inequality, walks.

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