

GRAPHS WITH ALL BUT TWO EIGENVALUES IN $[-2, 0]$

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Abstract

The eigenvalues of a graph are those of its adjacency matrix. Recently, Cioabă, Haemers and Vermette characterized all graphs with all but two eigenvalues equal to -2 and 0 . In this article, we extend their result by characterizing explicitly all graphs with all but two eigenvalues in the interval $[-2, 0]$. Also, we determine among them those that are determined by their spectrum.

Keywords: graph spectrum, complete multipartite graph, graph determined by its spectrum.

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