

## GRAPHS WITH ALL BUT TWO EIGENVALUES IN $[-2, 0]$

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### Abstract

The eigenvalues of a graph are those of its adjacency matrix. Recently, Cioabă, Haemers and Vermette characterized all graphs with all but two eigenvalues equal to  $-2$  and  $0$ . In this article, we extend their result by characterizing explicitly all graphs with all but two eigenvalues in the interval  $[-2, 0]$ . Also, we determine among them those that are determined by their spectrum.

**Keywords:** graph spectrum, complete multipartite graph, graph determined by its spectrum.

**2010 Mathematics Subject Classification:** 05C50.

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Received 7 April 2019  
Revised 11 October 2019  
Accepted 20 November 2019