

GRAPHS WITH CLUSTERS PERTURBED BY REGULAR GRAPHS— A_α -SPECTRUM AND APPLICATIONS

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Abstract

Given a graph G , its adjacency matrix $A(G)$ and its diagonal matrix of vertex degrees $D(G)$, consider the matrix $A_\alpha(G) = \alpha D(G) + (1 - \alpha)A(G)$, where $\alpha \in [0, 1]$. The A_α -spectrum of G is the multiset of eigenvalues of $A_\alpha(G)$ and these eigenvalues are the α -eigenvalues of G . A cluster in G is a pair of vertex subsets (C, S) , where C is a set of cardinality $|C| \geq 2$ of pairwise co-neighbor vertices sharing the same set S of $|S|$ neighbors. Assuming that G is connected and it has a cluster (C, S) , $G(H)$ is obtained from G and an r -regular graph H of order $|C|$ by identifying its vertices with the vertices in C , eigenvalues of $A_\alpha(G)$ and $A_\alpha(G(H))$ are deduced and if $A_\alpha(H)$ is positive semidefinite, then the i -th eigenvalue of $A_\alpha(G(H))$ is greater than or equal to i -th eigenvalue of $A_\alpha(G)$. These results are extended to graphs with several pairwise disjoint clusters $(C_1, S_1), \dots, (C_k, S_k)$. As an application, the effect on the energy, α -Estrada index and α -index of a graph G with clusters when the edges of regular graphs are added to G are analyzed. Finally, the A_α -spectrum of the corona product $G \circ H$ of a connected graph G and a regular graph H is determined.

Keywords: cluster, convex combination of matrices, A_α -spectrum, corona product of graphs.

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