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EXISTENCE OF REGULAR NUT GRAPHS FOR DEGREE AT MOST 11

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Dedicated to the memory of Slobodan Simić.

Abstract

A nut graph is a singular graph with one-dimensional kernel and corresponding eigenvector with no zero elements. The problem of determining the orders n for which d-regular nut graphs exist was recently posed by Gauci, Pisanski and Sciriha. These orders are known for $d \leq 4$. Here we solve the problem for all remaining cases $d \leq 11$ and determine the complete lists of all d-regular nut graphs of order n for small values of d and n. The existence or non-existence of small regular nut graphs is determined by a computer search. The main tool is a construction that produces, for any d-regular nut graph of order n, another d-regular nut graph of order $n+2d$. If we are given a sufficient number of d-regular nut graphs of consecutive orders, called seed graphs, this construction may be applied in such a way that the existence of all d-regular nut graphs of higher orders is established. For even d the orders n are indeed consecutive, while for odd d the orders n are consecutive even numbers. Furthermore, necessary conditions for combinations of order and degree for vertex-transitive nut graphs are derived.

Keywords: nut graph, core graph, regular graph, nullity.

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