

SIGNED COMPLETE GRAPHS WITH MAXIMUM INDEX

SAIEED AKBARI

Department of Mathematical Science
Sharif University of Technology, Tehran, Iran

e-mail: s_akbari@sharif.edu

SOUDABEH DALVANDI, FARIDEH HEYDARI

AND

MOHAMMAD MAGHASEDI

Department of Mathematics
Karaj Branch, Islamic Azad University, Karaj, Iran

e-mail: s.dalvandi@kiau.ac.ir
f.heydari@kiau.ac.ir
maghasedi@kiau.ac.ir

In the Memorial of Professor Slobodan Simić

Abstract

Let $\Gamma = (G, \sigma)$ be a signed graph, where G is the underlying simple graph and $\sigma : E(G) \rightarrow \{-, +\}$ is the sign function on the edges of G . The adjacency matrix of a signed graph has -1 or $+1$ for adjacent vertices, depending on the sign of the edges. It was conjectured that if Γ is a signed complete graph of order n with k negative edges, $k < n - 1$ and Γ has maximum index, then negative edges form $K_{1,k}$. In this paper, we prove this conjecture if we confine ourselves to all signed complete graphs of order n whose negative edges form a tree of order $k + 1$. A $[1, 2]$ -subgraph of G is a graph whose components are paths and cycles. Let Γ be a signed complete graph whose negative edges form a $[1, 2]$ -subgraph. We show that the eigenvalues of Γ satisfy the following inequalities:

$$-5 \leq \lambda_n \leq \dots \leq \lambda_2 \leq 3.$$

Keywords: signed graph, complete graph, index.

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