

A FEW EXAMPLES AND COUNTEREXAMPLES IN SPECTRAL GRAPH THEORY

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Dedicated to the memory of Professor Slobodan Simić.

Abstract

We present a small collection of examples and counterexamples for selected problems, mostly in spectral graph theory, that have occupied our minds over a number of years without being completely resolved.

Keywords: communicability distance, spectral radius, integral graph, second Zagreb index, Wiener index, estrada index, almost cospectral graphs, NEPS of graphs.

2010 Mathematics Subject Classification: 05C50.

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Received 26 April 2019
 Revised 26 August 2019
 Accepted 28 August 2019