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ON THE DISPLACEMENT OF EIGENVALUES WHEN REMOVING A TWIN VERTEX

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Dedicated to the memory of Slobodan K. Simić.

Abstract

Twin vertices of a graph have the same open neighbourhood. If they are not adjacent, then they are called duplicates and contribute the eigenvalue zero to the adjacency matrix. Otherwise they are termed co-duplicates, when they contribute -1 as an eigenvalue of the adjacency matrix. On removing a twin vertex from a graph, the spectrum of the adjacency matrix does not only lose the eigenvalue 0 or -1. The perturbation sends a rippling effect to the spectrum. The simple eigenvalues are displaced. We obtain a closed formula for the characteristic polynomial of a graph with twin vertices in terms of two polynomials associated with the perturbed graph. These are used to obtain estimates of the displacements in the spectrum caused by the perturbation.

Keywords: eigenvalues, perturbations, duplicate and co-duplicate vertices, threshold graph, nested split graph.

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