

## A SPECTRAL CHARACTERIZATION OF THE $s$ -CLIQUE EXTENSION OF THE TRIANGULAR GRAPHS

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*This paper is dedicated to the memory of Prof. Slobodan Simić.*

### Abstract

A regular graph is co-edge regular if there exists a constant  $\mu$  such that any two distinct and non-adjacent vertices have exactly  $\mu$  common neighbors. In this paper, we show that for integers  $s \geq 2$  and  $n$  large enough, any co-edge-regular graph which is cospectral with the  $s$ -clique extension of the triangular graph  $T(n)$  is exactly the  $s$ -clique extension of the triangular graph  $T(n)$ .

**Keywords:** co-edge-regular graph,  $s$ -clique extension, triangular graph.

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