Discussiones Mathematicae
Graph Theory 40 (2020) 495–524
doi:10.7151/dmgt.2271

ON THE DISTANCE SPECTRAL RADIUS OF TREES WITH GIVEN DEGREE SEQUENCE

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This paper is dedicated to the memory of Slobodan Simić.

Abstract

We consider the problem of maximizing the distance spectral radius and a slight generalization thereof among all trees with some prescribed degree sequence. We prove in particular that the maximum of the distance spectral radius has to be attained by a caterpillar for any given degree sequence. The same holds true for the terminal distance matrix. Moreover, we consider a generalized version of the reverse distance matrix and also study its spectral radius for trees with given degree sequence. We prove that the spectral radius is always maximized by a greedy tree. This implies several corollaries, among them a "reversed" version of a conjecture of Stevanović and Ilić. Our results parallel similar theorems for the Wiener index and other invariants.

Keywords: distance matrix, spectral radius, tree, degree sequence.

2010 Mathematics Subject Classification: 05C05, 05C50, 05C12.

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Received 30 April 2019
Revised 9 July 2019
Accepted 18 July 2019