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THE GENERAL POSITION PROBLEM ON KNESER GRAPHS AND ON SOME GRAPH OPERATIONS

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Abstract

A vertex subset S of a graph G is a general position set of G if no vertex of S lies on a geodesic between two other vertices of S. The cardinality of a largest general position set of G is the general position number (gp-number) $\mathrm{gp}(G)$ of G. The gp-number is determined for some families of Kneser graphs, in particular for $K(n, 2)$, $n \geq 4$, and $K(n, 3)$, $n \geq 9$. A sharp lower bound on the gp-number is proved for Cartesian products of graphs. The gp-number is also determined for joins of graphs, coronas over graphs, and line graphs of complete graphs.

Keywords: general position set, Kneser graphs, Cartesian product of graphs, corona over graphs, line graphs.

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References

[1] B.S. Anand, S.V. Ullas Chandran, M. Changat, S. Klavžar and E.J. Thomas, *Characterization of general position sets and its applications to cographs and bipartite graphs*, Appl. Math. Comput. **359** (2019) 84–89.
https://doi.org/10.1016/j.amc.2019.04.064

[2] G. Boruzanli Ekinci and J.B. Gauci, *The super-connectivity of Kneser graphs*, Discuss. Math. Graph Theory **39** (2019) 5–11.
https://doi.org/10.7151/dmgt.2051

[3] B. Brešar and M. Valencia-Pabon, *Independence number of products of Kneser graphs*, Discrete Math. **342** (2019) 1017–1027.
https://doi.org/10.1016/j.disc.2018.12.017

[4] K.N. Chadha and A.A. Kulkarni, *On independent cliques and linear complementarity problems* (2018).
arXiv:1811.09798

[5] H.E. Dudeney, Amusements in Mathematics (Nelson, Edinburgh, 1917).

[6] P. Erdős, C. Ko and R. Rado, *Intersection theorems for systems of finite sets*, Q. J. Math. **12** (1961) 313–320.
https://doi.org/10.1093/qmath/12.1.313

[7] V. Froese, I. Kanj, A. Nichterlein and R. Niedermeier, *Finding points in general position*, Internat. J. Comput. Geom. Appl. **27** (2017) 277–296.
https://doi.org/10.1142/S021819591750008X

[8] W. Imrich, S. Klavžar and D.F. Rall, Topics in Graph Theory: Graphs and Their Cartesian Product (A K Peters, New York 2008).
https://doi.org/10.1201/b10613

[9] M.M. Kanté, R.M. Sampaio, V.F. dos Santos and J.L. Szwarcfiter, *On the geodetic rank of a graph*, J. Comb. **8** (2017) 323–340.
https://doi.org/10.4310/JOC.2017.v8.n2.a5

[10] S. Klavžar, B. Patkós, G. Rus and I.G. Yero, *On general position sets in Cartesian grids* (2019).
arXiv:1907.04535v3

[11] S. Klavžar and I.G. Yero, *The general position problem and strong resolving graphs*, Open Math. **17** (2019) 1126–1135.
https://doi.org/10.1515/math-2019-0088

[12] C.Y. Ku and K.B. Wong, *On no-three-in-line problem on m-dimensional torus*, Graphs Combin. **34** (2018) 355–364.
https://doi.org/10.1007/s00373-018-1878-8

[13] J.H. van Lint and R.M. Wilson, A Course in Combinatorics (Cambridge University Press, Cambridge, 1992).
https://doi.org/10.1017/CBO9780511987045

[14] P. Manuel and S. Klavžar, *A general position problem in graph theory*, Bull. Aust. Math. Soc. **98** (2018) 177–187.
https://doi.org/10.1017/S0004972718000473

[15] P. Manuel and S. Klavžar, *The graph theory general position problem on some interconnection networks*, Fund. Inform. **163** (2018) 339–350.
https://doi.org/10.3233/FI-2018-1748

[16] A. Misiak, Z. Stępień, A. Szymaszkiewicz, L. Szymaszkiewicz and M. Zwierzchowski, *A note on the no-three-in-line problem on a torus*, Discrete Math. **339** (2016) 217–221.
https://doi.org/10.1016/j.disc.2015.08.006

[17] T. Mütze and P. Su, *Bipartite Kneser graphs are Hamiltonian*, Combinatorica **37** (2017) 1207–1219.
https://doi.org/10.1007/s00493-016-3434-6

[18] B. Patkós, *On the general position problem on Kneser graphs* (2019).
arXiv:1903.08056v2

[19] M. Payne and D.R. Wood, *On the general position subset selection problem*, SIAM J. Discrete Math. **27** (2013) 1727–1733.
https://doi.org/10.1137/120897493

[20] A. Por and D.R. Wood, *No-three-in-line-in-3D*, Algorithmica **47** (2007) 481–488.
https://doi.org/10.1007/s00453-006-0158-9

[21] S.V. Ullas Chandran and G. Jaya Parthasarathy, *The geodesic irredundant sets in graphs*, Int. J. Math. Combin. **4** (2016) 135–143.
https://doi.org/10.5281/zenodo.826834

[22] M. Valencia-Pabon and J.-C. Vera, *On the diameter of Kneser graphs*, Discrete Math. **305** (2005) 383–385.
https://doi.org/10.1016/j.disc.2005.10.001

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