

ON THE α -SPECTRAL RADIUS OF UNIFORM HYPERGRAPHS

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Abstract

For $0 \leq \alpha < 1$ and a uniform hypergraph G , the α -spectral radius of G is the largest H -eigenvalue of $\alpha\mathcal{D}(G) + (1-\alpha)\mathcal{A}(G)$, where $\mathcal{D}(G)$ and $\mathcal{A}(G)$ are the diagonal tensor of degrees and the adjacency tensor of G , respectively. We give upper bounds for the α -spectral radius of a uniform hypergraph, propose some transformations that increase the α -spectral radius, and determine the unique hypergraphs with maximum α -spectral radius in some classes of uniform hypergraphs.

Keywords: α -spectral radius, α -Perron vector, adjacency tensor, uniform hypergraph, extremal hypergraph.

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