

## ON THE $\alpha$ -SPECTRAL RADIUS OF UNIFORM HYPERGRAPHS

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### Abstract

For  $0 \leq \alpha < 1$  and a uniform hypergraph  $G$ , the  $\alpha$ -spectral radius of  $G$  is the largest  $H$ -eigenvalue of  $\alpha\mathcal{D}(G) + (1-\alpha)\mathcal{A}(G)$ , where  $\mathcal{D}(G)$  and  $\mathcal{A}(G)$  are the diagonal tensor of degrees and the adjacency tensor of  $G$ , respectively. We give upper bounds for the  $\alpha$ -spectral radius of a uniform hypergraph, propose some transformations that increase the  $\alpha$ -spectral radius, and determine the unique hypergraphs with maximum  $\alpha$ -spectral radius in some classes of uniform hypergraphs.

**Keywords:**  $\alpha$ -spectral radius,  $\alpha$ -Perron vector, adjacency tensor, uniform hypergraph, extremal hypergraph.

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