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FRACTIONAL REVIVAL OF THRESHOLD GRAPHS UNDER LAPLACIAN DYNAMICS

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This paper is dedicated to the memory of Slobodan Simić.

Abstract

We consider Laplacian fractional revival between two vertices of a graph X. Assume that it occurs at time τ between vertices 1 and 2. We prove that for the spectral decomposition $L = \sum_{r=0}^{q} \theta_r E_r$ of the Laplacian matrix L of X, for each $r = 0, 1, \ldots, q$, either $E_r e_1 = E_r e_2$, or $E_r e_1 = -E_r e_2$, depending on whether $e^{i\tau\theta_r}$ equals to 1 or not. That is to say, vertices 1 and 2 are strongly cospectral with respect to L. We give a characterization of the parameters of threshold graphs that allow for Laplacian fractional revival between two vertices; those graphs can be used to generate more graphs with Laplacian fractional revival. We also characterize threshold graphs that admit Laplacian fractional revival within a subset of more than two vertices. Throughout we rely on techniques from spectral graph theory.

Keywords: Laplacian matrix, spectral decomposition, quantum information transfer, fractional revival.

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