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ALGORITHMIC ASPECTS OF SECURE CONNECTED DOMINATION IN GRAPHS

Jakkepalli Pavan Kumar

and

P. Venkata Subba Reddy

Department of Computer Science and Engineering
National Institute of Technology
Warangal, Telangana, India

e-mail: jp.nitw@gmail.com
pvsr@nitw.ac.in

Abstract

Let $G = (V, E)$ be a simple, undirected and connected graph. A connected dominating set $S \subseteq V$ is a secure connected dominating set of G, if for each $u \in V \setminus S$, there exists $v \in S$ such that $(u, v) \in E$ and the set $(S \setminus \{v\}) \cup \{u\}$ is a connected dominating set of G. The minimum size of a secure connected dominating set of G denoted by $\gamma_{sc}(G)$, is called the secure connected domination number of G. Given a graph G and a positive integer k, the Secure Connected Domination (SCDM) problem is to check whether G has a secure connected dominating set of size at most k. In this paper, we prove that the SCDM problem is NP-complete for doubly chordal graphs, a subclass of chordal graphs. We investigate the complexity of this problem for some subclasses of bipartite graphs namely, star convex bipartite, comb convex bipartite, chordal bipartite and chain graphs. The Minimum Secure Connected Dominating Set (MSCDS) problem is to find a secure connected dominating set of minimum size in the input graph. We propose a $(\Delta(G)+1)$-approximation algorithm for MSCDS, where $\Delta(G)$ is the maximum degree of the input graph G and prove that MSCDS cannot be approximated within $(1-\epsilon)\ln(|V|)$ for any $\epsilon > 0$ unless $NP \subseteq DTIME\big(|V|^{O(\log\log|V|)}\big)$ even for bipartite graphs. Finally, we show that the MSCDS is APX-complete for graphs with $\Delta(G) = 4$.

Keywords: secure domination, complexity classes, tree-width, chordal graphs.

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