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# TOTAL DOMINATION IN GENERALIZED PRISMS AND A NEW DOMINATION INVARIANT

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**Abstract**

In this paper we complement recent studies on the total domination of prisms by considering generalized prisms, i.e., Cartesian products of an arbitrary graph and a complete graph. By introducing a new domination invariant on a graph $G$, called the *$k$-rainbow total domination number* and denoted by $\gamma_{k\mathrm{rt}}(G)$, it is shown that the problem of finding the total domination number of a generalized prism $G \,\square\, K_k$ is equivalent to an optimization problem of assigning subsets of $\{1, 2, \ldots, k\}$ to vertices of $G$. Various properties of the new domination invariant are presented, including, inter alia, that $\gamma_{k\mathrm{rt}}(G) = n$ for a nontrivial graph $G$ of order $n$ as soon as $k \geq 2\Delta(G)$. To prove the mentioned result as well as the closed formulas for the $k$-rainbow total domination number of paths and cycles for every $k$, a new weight-redistribution method is introduced, which serves as an efficient tool for establishing a lower bound for a domination invariant.

**Keywords:** domination, $k$-rainbow total domination, total domination.

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