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TOTAL DOMINATION IN GENERALIZED PRISMS AND A NEW DOMINATION INVARIANT

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Abstract

In this paper we complement recent studies on the total domination of prisms by considering generalized prisms, i.e., Cartesian products of an arbitrary graph and a complete graph. By introducing a new domination invariant on a graph G, called the *k-rainbow total domination number* and denoted by $\gamma_{k\mathrm{rt}}(G)$, it is shown that the problem of finding the total domination number of a generalized prism $G \,\square\, K_k$ is equivalent to an optimization problem of assigning subsets of $\{1, 2, \ldots, k\}$ to vertices of G. Various properties of the new domination invariant are presented, including, inter alia, that $\gamma_{k\mathrm{rt}}(G) = n$ for a nontrivial graph G of order n as soon as $k \geq 2\Delta(G)$. To prove the mentioned result as well as the closed formulas for the k-rainbow total domination number of paths and cycles for every k, a new weight-redistribution method is introduced, which serves as an efficient tool for establishing a lower bound for a domination invariant.

Keywords: domination, k-rainbow total domination, total domination.

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