Discussiones Mathematicae
Graph Theory 41 (2021) 1147–1163
https://doi.org/10.7151/dmgt.2245

EQUITABLE TOTAL COLORING OF CORONA OF CUBIC GRAPHS

Hanna Furmańczyk

Institute of Informatics
Faculty of Mathematics, Physics and Informatics
University of Gdańsk, Wita Stwosza 57, 80-308 Gdańsk, Poland

e-mail: hanna.furmanczyk@inf.ug.edu.pl

and

Rita Zuazua

Department of Mathematics, Faculty of Sciences
National Autonomous University of Mexico
Ciudad Universitaria, Coyoacan, 04510 Mexico, DF, Mexico

e-mail: ritazuazua@ciencias.unam.mx

Abstract

The minimum number of total independent partition sets of $V \cup E$ of a graph $G = (V, E)$ is called the *total chromatic number* of G, denoted by $\chi''(G)$. If the difference between cardinalities of any two total independent sets is at most one, then the minimum number of total independent partition sets of $V \cup E$ is called the *equitable total chromatic number*, and is denoted by $\chi''_{=}(G)$.

In this paper we consider equitable total coloring of coronas of cubic graphs, $G \circ H$. It turns out that independently on the values of equitable total chromatic number of factors G and H, equitable total chromatic number of corona $G \circ H$ is equal to $\Delta(G \circ H) + 1$. Thereby, we confirm Total Coloring Conjecture (TCC), posed by Behzad in 1964, and Equitable Total Coloring Conjecture (ETCC), posed by Wang in 2002, for coronas of cubic graphs. As a direct consequence we get that all coronas of cubic graphs are of Type 1.

Keywords: equitable coloring, total coloring, equitable total coloring, cubic graphs.

2010 Mathematics Subject Classification: 05C15, 05C76.

References

[1] M. Behzad, G. Chartrand and J.K. Cooper Jr, *The colour numbers of complete graphs*, J. Lond. Math. Soc. (2) **42** (1967) 226–228. https://doi.org/10.1112/jlms/s1-42.1.226

[2] B.L. Chen, K.W. Lih and P.L. Wu, *Equitable coloring and the maximum degree*, European J. Combin. **15** (1994) 443–447. https://doi.org/10.1006/eujc.1994.1047

[3] T. Chunling, L. Xiaohui, Y. Yuansheng and L. Zhihe, *Equitable total coloring of* $C_m \square C_n$, Discrete Appl. Math. **157** (2009) 596–601. https://doi.org/10.1016/j.dam.2008.08.030

[4] S. Dantas, C.M.H. de Figueiredo, G. Mazzuoccolo, M. Preissmann, V.F. dos Santos and D. Sasaki, *On the equitable total chromatic number of cubic graphs*, Discrete Appl. Math. **209** (2016) 84–91. https://doi.org/10.1016/j.dam.2015.10.013

[5] R. Frucht and F. Harary, *On the corona of two graphs*, Aequationes Math. **4** (1970) 322–325. https://doi.org/10.1007/BF01844162

[6] H.L. Fu, *Some results on equalized total coloring*, Congr. Numer. **102** (1994) 111–119.

[7] H. Furmańczyk, *Equitable coloring of graphs*, in: Graph Colorings, M. Kubale (Ed(s)), (Amer. Math. Soc. **352**, 2004) 35–53. https://doi.org/10.1090/conm/352/03

[8] H. Furmańczyk, *Equitable coloring of graph products*, Opuscula Math. **26** (2006) 31–44.

[9] H. Furmańczyk and M. Kubale, *Equitable coloring of corona products of cubic graphs is harder than ordinary coloring*, Ars Math. Contemp. **10** (2016) 333–347. https://doi.org/10.26493/1855-3974.687.99b

[10] H. Gui, W. Wang, Y. Wang and Z. Zhang, *Equitable total-coloring of subcubic graphs*, Discrete Appl. Math. **184** (2015) 167–170. https://doi.org/10.1016/j.dam.2014.11.014

[11] P. Hall, *On representatives of subsets*, J. London Math. Soc. (2) **10** (1935) 26– 30. https://doi.org/10.1112/jlms/s1-10.37.26

[12] W. Meyer, *Equitable coloring*, Amer. Math. Monthly **80** (1973) 920–922. https://doi.org/10.1080/00029890.1973.11993408

[13] A. Sánchez-Arroyo, *Determining the total colouring number is NP-hard*, Discrete Math. **78** (1989) 315–319. https://doi.org/10.1016/0012-365X(89)90187-8

[14] W.-F. Wang, *Equitable total coloring of graphs with maximum degree* 3, Graphs Combin. **18** (2002) 677–685. https://doi.org/10.1007/s003730200051

[15] N. Vijayaditya, *On total chromatic number of a graph*, J. London Math. Soc. **2** (1971) 405–408.
https://doi.org/10.1112/jlms/s2-3.3.405

[16] V.G. Vizing, *On an estimate of the chromatic class of a p-graph*, Metody Diskret. Analiz. **3** (1964) 25–30.

[17] W. Wang and K. Zhang, *Equitable colorings of line graphs and complete r-partite graphs*, Systems Sci. Math. Sci. **13** (2000) 190–194.

Received 8 June 2018
Revised 31 May 2019
Accepted 13 September 2019