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EQUITABLE TOTAL COLORING OF CORONA OF CUBIC GRAPHS

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Abstract

The minimum number of total independent partition sets of $V \cup E$ of a graph $G = (V, E)$ is called the *total chromatic number* of G, denoted by $\chi''(G)$. If the difference between cardinalities of any two total independent sets is at most one, then the minimum number of total independent partition sets of $V \cup E$ is called the *equitable total chromatic number*, and is denoted by $\chi''_{=}(G)$.

In this paper we consider equitable total coloring of coronas of cubic graphs, $G \circ H$. It turns out that independently on the values of equitable total chromatic number of factors G and H, equitable total chromatic number of corona $G \circ H$ is equal to $\Delta(G \circ H) + 1$. Thereby, we confirm Total Coloring Conjecture (TCC), posed by Behzad in 1964, and Equitable Total Coloring Conjecture (ETCC), posed by Wang in 2002, for coronas of cubic graphs. As a direct consequence we get that all coronas of cubic graphs are of Type 1.

Keywords: equitable coloring, total coloring, equitable total coloring, cubic graphs.

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