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ON THE ISOMETRIC PATH PARTITION PROBLEM

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Abstract

The isometric path cover (partition) problem of a graph consists of finding a minimum set of isometric paths which cover (partition) the vertex set of the graph. The isometric path cover (partition) number of a graph is the cardinality of a minimum isometric path cover (partition). We prove that the isometric path partition problem and the isometric k-path partition problem for $k \geq 3$ are NP-complete on general graphs. Fisher and Fitzpatrick in [*The isometric number of a graph*, J. Combin. Math. Combin. Comput. 38 (2001) 97–110] have shown that the isometric path cover number of the $(r \times r)$-dimensional grid is $\lceil 2r/3 \rceil$. We show that the isometric path cover (partition) number of the $(r \times s)$-dimensional grid is s when $r \geq s(s-1)$. We establish that the isometric path cover (partition) number of the $(r \times r)$-dimensional torus is r when r is even and is either r or $r+1$ when r is odd. Then, we demonstrate that the isometric path cover (partition) number of an r-dimensional Benes network is 2^r. In addition, we provide partial solutions for the isometric path cover (partition) problems for cylinder and multi-dimensional grids. We apply two different techniques to achieve these results.

Keywords: path cover problem, isometric path partition problem, isometric path cover problem, multi-dimensional grids, cylinder, torus.

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