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A NOTE ON THE EQUITABLE CHOOSABILITY OF COMPLETE BIPARTITE GRAPHS

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Abstract

In 2003 Kostochka, Pelsmajer, and West introduced a list analogue of equitable coloring called equitable choosability. A k-assignment, L, for a graph G assigns a list, $L(v)$, of k available colors to each $v \in V(G)$, and an equitable L-coloring of G is a proper coloring, f, of G such that $f(v) \in L(v)$ for each $v \in V(G)$ and each color class of f has size at most $\lceil |V(G)|/k \rceil$. Graph G is said to be equitably k-choosable if an equitable L-coloring of G exists whenever L is a k-assignment for G. In this note we study the equitable choosability of complete bipartite graphs. A result of Kostochka, Pelsmajer, and West implies $K_{n,m}$ is equitably k-choosable if $k \geq \max\{n, m\}$ provided $K_{n,m} \neq K_{2l+1,2l+1}$. We prove $K_{n,m}$ is equitably k-choosable if $m \leq \lceil (m+n)/k \rceil (k-n)$ which gives $K_{n,m}$ is equitably k-choosable for certain k satisfying $k < \max\{n, m\}$. We also give a complete characterization of the equitable choosability of complete bipartite graphs that have a partite set of size at most 2.

Keywords: graph coloring, equitable coloring, list coloring, equitable choosability.

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