

NON-1-PLANARITY OF LEXICOGRAPHIC PRODUCTS OF GRAPHS

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Abstract

In this paper, we show the non-1-planarity of the lexicographic product of a theta graph and K_2 . This result completes the proof of the conjecture that a graph $G \circ K_2$ is 1-planar if and only if G has no edge belonging to two cycles.

Keywords: 1-planar graph, lexicographic product.

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