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ON EDGE H-IRREGULARITY STRENGTHS OF SOME GRAPHS

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Abstract

For a graph G an *edge-covering* of G is a family of subgraphs $H_1, H_2, \ldots, H_t$ such that each edge of $E(G)$ belongs to at least one of the subgraphs H_i, $i = 1, 2, \ldots, t$. In this case we say that G admits an $(H_1, H_2, \ldots, H_t)$-(*edge*) *covering*. An H-*covering* of graph G is an $(H_1, H_2, \ldots, H_t)$-(*edge*) *covering* in which every subgraph H_i is isomorphic to a given graph H.

Let G be a graph admitting H-covering. An edge k-labeling $\alpha : E(G) \to \{1, 2, \ldots, k\}$ is called an H-*irregular edge* k-*labeling* of the graph G if for every two different subgraphs H' and H'' isomorphic to H their weights

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$wt_\alpha(H')$ and $wt_\alpha(H'')$ are distinct. The weight of a subgraph H under an edge k-labeling α is the sum of labels of edges belonging to H. The *edge H-irregularity strength* of a graph G, denoted by $\mathrm{ehs}(G, H)$, is the smallest integer k such that G has an H-irregular edge k-labeling.

In this paper we determine the exact values of $\mathrm{ehs}(G, H)$ for prisms, antiprisms, triangular ladders, diagonal ladders, wheels and gear graphs. Moreover the subgraph H is isomorphic to only C_4, C_3 and K_4.

Keywords: H-irregular edge labeling, edge H-irregularity strength, prism, antiprism, triangular ladder, diagonal ladder, wheel, gear graph.

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