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$\mathbb{Z}_2 \times \mathbb{Z}_2$-CORDIAL CYCLE-FREE HYPERGRAPHS

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Abstract

Hovey introduced A-cordial labelings as a generalization of cordial and harmonious labelings [7]. If A is an Abelian group, then a labeling $f\colon V(G) \to A$ of the vertices of some graph G induces an edge labeling on G; the edge uv receives the label $f(u)+f(v)$. A graph G is A-cordial if there is a vertex-labeling such that (1) the vertex label classes differ in size by at most one and (2) the induced edge label classes differ in size by at most one.

The problem of A-cordial labelings of graphs can be naturally extended for hypergraphs. It was shown that not every 2-uniform hypertree (i.e., tree) admits a $\mathbb{Z}_2 \times \mathbb{Z}_2$-cordial labeling [8]. The situation changes if we consider p-uniform hypertrees for a bigger p. We prove that a p-uniform hypertree is $\mathbb{Z}_2 \times \mathbb{Z}_2$-cordial for any $p > 2$, and so is every path hypergraph in which all edges have size at least 3. The property is not valid universally in the class of hypergraphs of maximum degree 1, for which we provide a necessary and sufficient condition.

Keywords: V_4-cordial graph, hypergraph, labeling of hypergraph, hypertree.

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