

## RESTRAINED DOMINATION IN SELF-COMPLEMENTARY GRAPHS

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### Abstract

A self-complementary graph is a graph isomorphic to its complement. A set  $S$  of vertices in a graph  $G$  is a restrained dominating set if every vertex in  $V(G) \setminus S$  is adjacent to a vertex in  $S$  and to a vertex in  $V(G) \setminus S$ . The restrained domination number of a graph  $G$  is the minimum cardinality of a restrained dominating set of  $G$ . In this paper, we study restrained domination in self-complementary graphs. In particular, we characterize the self-complementary graphs having equal domination and restrained domination numbers.

**Keywords:** domination, complement, restrained domination, self-complementary graph.

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