

NEIGHBOR PRODUCT DISTINGUISHING TOTAL  
COLORINGS OF PLANAR GRAPHS WITH  
MAXIMUM DEGREE AT LEAST TEN<sup>1</sup>

AIJUN DONG

*School of Data and Computer Science*  
*Shandong Women's University*  
*Jinan, 250300, P.R. China*

**e-mail:** dongaijun@mail.sdu.edu.cn

AND

TONG LI

*School of Mathematics*  
*Shandong University*  
*Jinan 250100, P.R. China*

**e-mail:** tli@sdu.edu.cn

**Abstract**

A proper  $[k]$ -total coloring  $c$  of a graph  $G$  is a proper total coloring  $c$  of  $G$  using colors of the set  $[k] = \{1, 2, \dots, k\}$ . Let  $p(u)$  denote the product of the color on a vertex  $u$  and colors on all the edges incident with  $u$ . For each edge  $uv \in E(G)$ , if  $p(u) \neq p(v)$ , then we say the coloring  $c$  distinguishes adjacent vertices by product and call it a neighbor product distinguishing  $k$ -total coloring of  $G$ . By  $\chi''_{\Pi}(G)$ , we denote the smallest value of  $k$  in such a coloring of  $G$ . It has been conjectured by Li *et al.* that  $\Delta(G) + 3$  colors enable the existence of a neighbor product distinguishing total coloring. In this paper, by applying the Combinatorial Nullstellensatz, we obtain that the conjecture holds for planar graph with  $\Delta(G) \geq 10$ . Moreover, for planar graph  $G$  with  $\Delta(G) \geq 11$ , it is neighbor product distinguishing  $(\Delta(G) + 2)$ -total colorable, and the upper bound  $\Delta(G) + 2$  is tight.

**Keywords:** total coloring, neighbor product distinguishing coloring, planar graph.

**2010 Mathematics Subject Classification:** 05C15.

---

<sup>1</sup>This work was supported by the National Natural Science Foundation of China (Grant No. 61773015). It was also supported by China Postdoctoral Science Foundation Funded Project (Grant No.2014M561909); the Nature Science Foundation of Shandong Province of China (Grant No. ZR2014AM028, ZR2017LEM014).

## REFERENCES

- [1] N. Alon, *Combinatorial Nullstellensatz*, Combin. Probab. Comput. **8** (1999) 7–29.  
<https://doi.org/10.1017/S0963548398003411>
- [2] J.A. Bondy and U.S.R. Murty, *Graph Theory with Applications* (North-Holland, New York, 1976).
- [3] X.E. Chen, *On the adjacent vertex distinguishing total coloring numbers of graphs with  $\Delta = 3$* , Discrete Math. **308** (2008) 4003–4007.  
<https://doi.org/10.1016/j.disc.2007.07.091>
- [4] X.H. Cheng, G.H. Wang and J.L. Wu, *The adjacent vertex distinguishing total chromatic numbers of planar graphs  $\Delta = 10$* , J. Comb. Optim. **34** (2017) 383–397.  
<https://doi.org/10.1007/s10878-016-9995-x>
- [5] X.H. Cheng and J.L. Wu, *The adjacent vertex distinguishing total choosability of planar graphs with maximum degree at least eleven*, J. Comb. Optim. **35** (2018) 1–13.  
<https://doi.org/10.1007/s10878-017-0149-6>
- [6] L.H. Ding, G.H. Wang and G.Y. Yan, *Neighbor sum distinguishing total colorings via the Combinatorial Nullstellensatz*, Sci. China Math. **57** (2014) 1875–1882.  
<https://doi.org/10.1007/s11425-014-4796-0>
- [7] L.H. Ding, G.H. Wang, J.L. Wu and J.G. Yu, *Neighbor sum (set) distinguishing total choosability via the Combinatorial Nullstellensatz*, Graphs Combin. **33** (2017) 885–900.  
<https://doi.org/10.1007/s00373-017-1806-3>
- [8] S. Ge, J.G. Li and C.Q. Xu, *Neighbor sum distinguishing total coloring of planar graphs without 5-cycles*, Theoret. Comput. Sci. **689** (2017) 169–175.  
<https://doi.org/10.1016/j.tcs.2017.05.037>
- [9] D.J. Huang and W.F. Wang, *Adjacent vertex distinguishing total coloring of planar graphs with large maximum degree*, Sci. Sin. Math. **42** (2012) 151–164 (in Chinese).  
<https://doi.org/10.1360/012011-359>
- [10] T. Li, C.Q. Qu, G.H. Wang and X.W. Yu, *Neighbor product distinguishing total colorings*, J. Comb. Optim. **33** (2017) 237–253.  
<https://doi.org/10.1007/s10878-015-9952-0>
- [11] H.L. Li, B.Q. Liu and G.H. Wang, *Neighbor sum distinguishing total colorings of  $K_4$ -minor free graphs*, Front. Math. China **8** (2013) 1351–1366.  
<https://doi.org/10.1007/s11464-013-0322-x>
- [12] H.L. Li, L.H. Ding, B.Q. Liu and G.H. Wang, *Neighbor sum distinguishing total colorings of planar graphs*, J. Comb. Optim. **30** (2015) 675–688.  
<https://doi.org/10.1007/s10878-013-9660-6>
- [13] Y. Lu, J.A. Li, R. Luo and Z.K. Miao, *Adjacent vertex distinguishing total coloring of graphs with maximum degree 4*, Discrete Math. **340** (2017) 119–123.  
<https://doi.org/10.1016/j.disc.2016.07.011>

- [14] Y. Lu, C.D. Xu and Z.K. Miao, *Neighbor sum distinguishing list total coloring of subcubic graphs*, J. Comb. Optim. **35** (2018) 778–793.  
<https://doi.org/10.1007/s10878-017-0239-5>
- [15] C.Q. Qu, G.H. Wang, J.L. Wu and X.W. Yu, *On the neighbor sum distinguishing total coloring of planar graphs*, Theoret. Comput. Sci. **609** (2016) 162–170.  
<https://doi.org/10.1016/j.tcs.2015.09.017>
- [16] H.J. Song and C.Q. Xu, *Neighbor sum distinguishing total coloring of planar graphs without 4-cycles*, J. Comb. Optim. **34** (2017) 1147–1158.  
<https://doi.org/10.1007/s10878-017-0137-x>
- [17] L. Sun, X.H. Cheng and J.L. Wu, *The adjacent vertex distinguishing total coloring of planar graphs without adjacent 4-cycles*, J. Comb. Optim. **33** (2017) 779–790.  
<https://doi.org/10.1007/s10878-016-0004-1>
- [18] H.Y. Wang, *On the adjacent vertex-distinguishing total chromatic numbers of the graphs with  $\Delta(G) = 3$* , J. Comb. Optim. **14** (2007) 87–109.  
<https://doi.org/10.1007/s10878-006-9038-0>
- [19] H.Y. Wang, *The adjacent vertex-distinguishing total chromatic number of 1-tree*, Ars Combin. **91** (2009) 183–192.
- [20] Y.Q. Wang and W.F. Wang, *Adjacent vertex distinguishing total colorings of outer-planar graphs*, J. Comb. Optim. **19** (2010) 123–133.  
<https://doi.org/10.1007/s10878-008-9165-x>
- [21] W.F. Wang and P. Wang, *On adjacent-vertex-distinguishing total coloring of  $K_4$ -minor free graphs*, Sci. China Ser. A Math. **39** (2009) 1462–1472 (in Chinese).  
<https://doi.org/10.1360/za2009-39-12-1462>
- [22] W.F. Wang and Y.Q. Wang, *Adjacent vertex distinguishing total coloring of graphs with lower average degree*, Taiwanese J. Math. **12** (2008) 979–990.  
<https://doi.org/10.11650/twjm/1500404991>
- [23] W.F. Wang and D.J. Huang, *The adjacent vertex distinguishing total coloring of planar graphs*, J. Comb. Optim. **27** (2014) 379–396.  
<https://doi.org/10.1007/s10878-012-9527-2>
- [24] D.L. Yang, L. Sun, X.W. Yu, J.L. Wu and S. Zhou, *Neighbor sum distinguishing total chromatic number of planar graphs with maximum degree 10*, Appl. Math. Comput. **314** (2017) 456–468.  
<https://doi.org/10.1016/j.amc.2017.06.002>
- [25] Z.F. Zhang, X.E. Chen, J.W. Li, B. Yao, X.Z. Lu and J.F. Wang, *On adjacent-vertex-distinguishing total coloring of graphs*, Sci. China Math. **48** (2005) 289–299.  
<https://doi.org/10.1360/03ys0207>

Received 11 October 2018

Revised 27 March 2019

Accepted 27 March 2019