Discussiones Mathematicae
Graph Theory 41 (2021) 981–999
https://doi.org/10.7151/dmgt.2221

NEIGHBOR PRODUCT DISTINGUISHING TOTAL COLORINGS OF PLANAR GRAPHS WITH MAXIMUM DEGREE AT LEAST TEN [1]

Aijun Dong

School of Data and Computer Science
Shandong Women's University
Jinan, 250300, P.R. China

e-mail: dongaijun@mail.sdu.edu.cn

AND

Tong Li

School of Mathematics
Shandong University
Jinan 250100, P.R. China

e-mail: tli@sdu.edu.cn

Abstract

A proper $[k]$-total coloring c of a graph G is a proper total coloring c of G using colors of the set $[k] = \{1, 2, \ldots, k\}$. Let $p(u)$ denote the product of the color on a vertex u and colors on all the edges incident with u. For each edge $uv \in E(G)$, if $p(u) \neq p(v)$, then we say the coloring c distinguishes adjacent vertices by product and call it a neighbor product distinguishing k-total coloring of G. By $\chi''_{\prod}(G)$, we denote the smallest value of k in such a coloring of G. It has been conjectured by Li *et al.* that $\Delta(G) + 3$ colors enable the existence of a neighbor product distinguishing total coloring. In this paper, by applying the Combinatorial Nullstellensatz, we obtain that the conjecture holds for planar graph with $\Delta(G) \geq 10$. Moreover, for planar graph G with $\Delta(G) \geq 11$, it is neighbor product distinguishing $(\Delta(G) + 2)$-total colorable, and the upper bound $\Delta(G) + 2$ is tight.

Keywords: total coloring, neighbor product distinguishing coloring, planar graph.

2010 Mathematics Subject Classification: 05C15.

[1]This work was supported by the National Natural Science Foundation of China (Grant No. 61773015). It was also supported by China Postdoctoral Science Foundation Funded Project (Grant No.2014M561909); the Nature Science Foundation of Shandong Province of China (Grant No. ZR2014AM028, ZR2017LEM014).

References

[1] N. Alon, *Combinatorial Nullstellensatz*, Combin. Probab. Comput. **8** (1999) 7–29. https://doi.org/10.1017/S0963548398003411

[2] J.A. Bondy and U.S.R. Murty, Graph Theory with Applications (North-Holland, New York, 1976).

[3] X.E. Chen, *On the adjacent vertex distinguishing total coloring numbers of graphs with* $\Delta = 3$, Discrete Math. **308** (2008) 4003–4007. https://doi.org/10.1016/j.disc.2007.07.091

[4] X.H. Cheng, G.H. Wang and J.L. Wu, *The adjacent vertex distinguishing total chromatic numbers of planar graphs* $\Delta = 10$, J. Comb. Optim. **34** (2017) 383–397. https://doi.org/10.1007/s10878-016-9995-x

[5] X.H. Cheng and J.L. Wu, *The adjacent vertex distinguishing total choosability of planar graphs with maximum degree at least eleven*, J. Comb. Optim. **35** (2018) 1–13. https://doi.org/10.1007/s10878-017-0149-6

[6] L.H. Ding, G.H. Wang and G.Y. Yan, *Neighbor sum distinguishing total colorings via the Combinatorial Nullstellensatz*, Sci. China Math. **57** (2014) 1875–1882. https://doi.org/10.1007/s11425-014-4796-0

[7] L.H. Ding, G.H. Wang, J.L. Wu and J.G. Yu, *Neighbor sum (set) distinguishing total choosability via the Combinatorial Nullstellensatz*, Graphs Combin. **33** (2017) 885–900. https://doi.org/10.1007/s00373-017-1806-3

[8] S. Ge, J.G. Li and C.Q. Xu, *Neighbor sum distinguishing total coloring of planar graphs without* 5*-cycles*, Theoret. Comput. Sci. **689** (2017) 169–175. https://doi.org/10.1016/j.tcs.2017.05.037

[9] D.J. Huang and W.F. Wang, *Adjacent vertex distinguishing total coloring of planar graphs with large maximum degree*, Sci. Sin. Math. **42** (2012) 151–164 (in Chinese). https://doi.org/10.1360/012011-359

[10] T. Li, C.Q. Qu, G.H. Wang and X.W. Yu, *Neighbor product distinguishing total colorings*, J. Comb. Optim. **33** (2017) 237–253. https://doi.org/10.1007/s10878-015-9952-0

[11] H.L. Li, B.Q. Liu and G.H. Wang, *Neighbor sum distinguishing total colorings of* K_4*-minor free graphs*, Front. Math. China **8** (2013) 1351–1366. https://doi.org/10.1007/s11464-013-0322-x

[12] H.L. Li, L.H. Ding, B.Q. Liu and G.H. Wang, *Neighbor sum distinguishing total colorings of planar graphs*, J. Comb. Optim. **30** (2015) 675–688. https://doi.org/10.1007/s10878-013-9660-6

[13] Y. Lu, J.A. Li, R. Luo and Z.K. Miao, *Adjacent vertex distinguishing total coloring of graphs with maximum degree* 4, Discrete Math. **340** (2017) 119–123. https://doi.org/10.1016/j.disc.2016.07.011

[14] Y. Lu, C.D. Xu and Z.K. Miao, *Neighbor sum distinguishing list total coloring of subcubic graphs*, J. Comb. Optim. **35** (2018) 778–793.
https://doi.org/10.1007/s10878-017-0239-5

[15] C.Q. Qu, G.H. Wang, J.L. Wu and X.W. Yu, *On the neighbor sum distinguishing total coloring of planar graphs*, Theoret. Comput. Sci. **609** (2016) 162–170.
https://doi.org/10.1016/j.tcs.2015.09.017

[16] H.J. Song and C.Q. Xu, *Neighbor sum distinguishing total coloring of planar graphs without* 4*-cycles*, J. Comb. Optim. **34** (2017) 1147–1158.
https://doi.org/10.1007/s10878-017-0137-x

[17] L. Sun, X.H. Cheng and J.L. Wu, *The adjacent vertex distinguishing total coloring of planar graphs without adjacent* 4*-cycles*, J. Comb. Optim. **33** (2017) 779–790.
https://doi.org/10.1007/s10878-016-0004-1

[18] H.Y. Wang, *On the adjacent vertex-distinguishing total chromatic numbers of the graphs with* $\Delta(G) = 3$, J. Comb. Optim. **14** (2007) 87–109.
https://doi.org/10.1007/s10878-006-9038-0

[19] H.Y. Wang, *The adjacent vertex-distinguishing total chromatic number of* 1*-tree*, Ars Combin. **91** (2009) 183–192.

[20] Y.Q. Wang and W.F. Wang, *Adjacent vertex distinguishing total colorings of outer-planar graphs*, J. Comb. Optim. **19** (2010) 123–133.
https://doi.org/10.1007/s10878-008-9165-x

[21] W.F. Wang and P. Wang, *On adjacent-vertex-distinguishing total coloring of* K_4*-minor free graphs*, Sci. China Ser. A Math. **39** (2009) 1462–1472 (in Chinese).
https://doi.org/10.1360/za2009-39-12-1462

[22] W F. Wang and Y.Q. Wang, *Adjacent vertex distinguishing total coloring of graphs with lower average degree*, Taiwanese J. Math. **12** (2008) 979–990.
https://doi.org/10.11650/twjm/1500404991

[23] W.F. Wang and D.J. Huang, *The adjacent vertex distinguishing total coloring of planar graphs*, J. Comb. Optim. **27** (2014) 379–396.
https://doi.org/10.1007/s10878-012-9527-2

[24] D.L. Yang, L. Sun, X.W. Yu, J.L. Wu and S. Zhou, *Neighbor sum distinguishing total chromatic number of planar graphs with maximum degree* 10, Appl. Math. Comput. **314** (2017) 456–468.
https://doi.org/10.1016/j.amc.2017.06.002

[25] Z.F. Zhang, X.E. Chen, J.W. Li, B. Yao, X.Z. Lu and J.F. Wang, *On adjacent-vertex-distinguishing total coloring of graphs*, Sci. China Math. **48** (2005) 289–299.
https://doi.org/10.1360/03ys0207

Received 11 October 2018
Revised 27 March 2019
Accepted 27 March 2019